Histogram Equalisation
The algorithm

- Given an image as below, derive the intensity mapping that will as best as possible equalise the image histogram. The image histogram is as shown on the right. There are 8 possible grey scale levels from 0 to 7.

4	4	4	4	4
3	4	5	4	3
3	5	5	5	3
3	4	5	4	3
4	4	4	4	4

I	0	1	2	3	4	5	6	7
f(I)	0	0	0	6	14	5	0	0

Step 1: calculate the cumulative frequency distribution

I	0	1	2	3	4	5	6	7
f(I)	0	0	0	6	14	5	0	0
Cuf	0	0	0	6	20	25	25	25

Idea is to derive an intensity mapping that will make the CuF turn into
a straight ramp.

Step 2: Compare with the CuF of an equalised histogram

I	0	1	2	3	4	5	6	7
f(I)	0	0	0	6	14	5	0	0
Cuf	0	0	0	6	20	25	25	25
Feq	3	3	3	4	3	3	3	3

In this case the Equalised (ideal) histogram needs to have 25/8 pels in each bin = 3.125 but only integer numbers of pels are possible (its frequency after all). So I've added in one to make it ok.

Step 2: Compare with the CuF of an equalised histogram

I	0	1	2	3	4	5	6	7
f(I)	0	0	0	6	14	5	0	0
Cuf	0	0	0	6	20	25	25	25
Feq	3	3	3	4	3	3	3	3
CuFeq	3	6	9	13	16	19	22	25

Step 2: Design the mapping

I	0	1	2	3	4	5	6	7
f(I)	0	0	0	6	14	5	0	0
Cuf	0	0	0	6	20	25	25	25
Feq	3	3	3	4	3	3	3	3
CuFeq	3	6	9	13	16	19	22	25

For each intensity in the original image, find an intensity in the transformed image that has as close as possible, the same amount of Cumulative frequency.

Step 2: Design the mapping

I	0	1	2	3	4	5	6	7
f(I)	0	0	0	6	14	5	0	0
Cuf	0	0	0	6	20	25	0	0
Feq	3	3	3	4	3	3	3	3
CuFeq	3	6	9	13	16	19	22	25

So for intensity 0, this has Cuf 0, and the closest in the xformed imag
Is a CuFreq of 3. So Intensity 0 in input maps to intensity 0 in output

Step 2: Design the mapping

I	0	1	2	3	4	5	6	7
f(I)	0	0	0	6	14	5	0	0
Cuf	0	0	0	6	20	25	25	25
Feq	3	3	3	4	3	3	3	3
CuFeq	3	6	9	13	16	19	22	25

Intensity 1 in input maps to Intensity 0 in output.
[Sometimes you can decide not to map into previously used intensities]

Step 2: Design the mapping

I	0	1	2	3	4	5	6	7
f(I)	0	0	0	6	14	5	0	0
Cuf	0	0	0	6	20	25	25	25
Feq	3	3	3	4	3	3	3	3
CuFeq	3	6	9	13	16	19	22	25

Input I	Output I
0	0
1	0
2	0
3	1

For each intensity in the original image, find an intensity in the transformed image that has as close as possible, the same amount of Cumulative frequency.

Step 2: Design the mapping

I	0	1	2	3	4	5	6	7
f(I)	0	0	0	6	14	5	0	0
Cuf	0	0	0	6	20	25	25	25
Feq	3	3	3	4	3	3	3	3
CuFeq	3	6	9	13	16	19	22	25

Input I	Output I
0	0
1	0
2	0
3	1
4	5

4 -> 5

Step 2: Design the mapping

I	0	1	2	3	4	5	6	7
f(I)	0	0	0	6	14	5	0	0
Cuf	0	0	0	6	20	25	25	25
Feq	3	3	3	4	3	3	3	3
CuFeq	3	6	9	13	16	19	22	25

Input I	Output I
0	0
1	0
2	0
3	1
4	5
5	7

5 ->7

Step 2: Design the mapping

I	0	1	2	3	4	5	6	7
f(I)	0	0	0	6	14	5	0	0
Cuf	0	0	0	6	20	25	25	25
Feq	3	3	3	4	3	3	3	3
CuFeq	3	6	9	13	16	19	22	25

Input I	Output I
0	0
1	0
2	0
3	1
4	5
5	7
6	7
7	7

And so on …

The new histogram

I	0	1	2	3	4	5	6	7
f(I)	0	0	0	6	14	5	0	0
Cuf	0	0	0	6	20	25	25	25
Feq	3	3	3	4	3	3	3	3
CuFeq	3	6	9	13	16	19	22	25
F(EQ)	0	6	0	0	0	14	0	5
Cu(EQ)	0	6	6	6	6	20	20	25

Input I	Output I
0	0
1	0
2	0
3	1
4	5
5	7
6	7
7	7

Output histogram only uses
4 bins!

The result

4	4	4	4	4
3	4	5	4	3
3	5	5	5	3
3	4	5	4	3
4	4	4	4	4

Original

Input I	Output I
0	0
1	0
2	0
3	1
4	5
5	7
6	7
7	7

5	5	5	5	5
1	5	7	5	1
1	7	7	7	1
1	5	7	5	1
5	5	5	5	5

Equalised