TRANSIENT CIRCUIT RESPONSE II

- Given an RC circuit in which the capacitor is completely discharged we have shown that when a voltage is suddenly applied e.g. via a switch the voltage across the capacitor exponentially approaches the voltage of the source.

- What happens if a non-constant voltage source is applied? Suppose the voltage begins at 0V, increases instantaneously to \( v_{in} \) Volts and then shortly thereafter drops to 0 volts? This is an example of the representation of a single *bit* in a bit stream of information e.g. video, Cd Audio etc.

- This will lead to an explanation for the numerous communications standards that exist.

- We will now look at the system response to this single *pulse* input in different ways.
1 First principles

The defining differential equation remains the same
\[ v_i(t) = RC \frac{dv_o(t)}{dt} + v_o(t) \]

The pulse has a width of \( T \) seconds, and 1 Volt amplitude. It is given that the capacitor is completely discharged before the pulse is input. The sequence of events is as follows

1. The input voltage rises suddenly to \( v_{in} = 1 \) Volt.
2. While the input voltage remains steady, the capacitor charges up and the voltage across its terminals increases steadily according to
\[ v_o = 1 - \exp\left(-\frac{t}{RC}\right) \]  
which we found out in the last handout
3. At \( t = T \), the input voltage drops to zero instantaneously, and the capacitor discharges through the resistor. Using the expression above, we know what the initial voltage is across the capacitor when the voltage suddenly drops to zero, hence we can calculate the response.
1.1 Capacitor discharge

Initial condition: \( v_o = A \) Volts, note that now we are starting \( t \) at the end of the pulse. So \( t = 0 \) for this calculation is actually \( t = T \) secs when the pulse is considered in its entirety.

\[
v_o(t) = RC \frac{dv_o(t)}{dt}
\]

\[
\Rightarrow V_o(s) = RC [sV_o(s) - v_o(0)]
\]

\[
V_o(s) =
\]

\[
V_o(s) [1 - RCs] =
\]

\[
V_o(s) = \left( \frac{-RC}{1 - RCs} \right)
\]

\[
= \left( \frac{-RC}{1 - RCs} \right)
\]

\[
\Rightarrow v_o(t) =
\]

Remember now \( A = 1 - e^{-T/RC} \) Volts

So: \( v_o(t) =
\]

But this episode actually occurs from \( t = T \) secs! so we must shift the time axis accordingly to give

\[
v_o(t) = e^{-(t-T)/(RC)} \left[ 1 - e^{-T/(RC)} \right]
\]
1.2 The whole response to a pulse

Response is therefore charging response up till $t = T$ secs, then discharge response thereafter i.e.

$$v_o(t) = \begin{cases} 
1 - e^{\frac{-t}{RC}} \\
 e^{-t/RC} \left[ e^{T/RC} - 1 \right]
\end{cases}$$
2 An easier way: using superposition and linearity

\[ u(t) \quad \text{cable} \]
\[ -u(t-T) \quad \text{cable} \]
\[ \text{cable} \]
3 The problem with channels . . .

- In communications a bit of 1 (ONE) is represented by a high voltage level and a bit of 0 (ZERO) is represented by a low voltage level (or a -ve level).

- A bit stream of information therefore looks like a bunch of pulses

- To transmit a pulse ... you just hook a wire up between the transmitter and the receiver. You put a pulse in one end and hope to get out a pulse at the other.

- Most cables for communications (twisted pair, coax . . . ) can be reasonably approximated by an RC cct.

- You have seen that this RC model will cause the pulse to become smeared at the other end. Sharp edges turn into smooth edges.

- To tell the difference between a one and a zero . . . the only way is to measure the voltage level. For instance, anything above 0.5 V say, is a ONE 1, while anything below that is a 0, ZERO.

- Generally, noise from spurious sources always corrupts the signal.

- So the effect of smearing, is to reduce the separability of a ONE from a ZERO especially in the presence of noise.

- Even worse than that . . . pulse widths have to be wide enough for them to pass through the channel unaffected.

- But the channel has memory . . . i.e. the response to the current bit is affected by previous bits. You can see this from the fact that the channel response does not decay quickly enough if the RC time constant is too large.

- What happens for different kinds of combinations of 1’s and 0’s? The smearing effect gets even worse. Previous 1’s can push up the levels of subsequent 0’s and vice versa . . . aargh! Called Intersymbol Interference
4 A Design Example

Cable connecting a sending (driving) circuit to a receiving circuit can be modelled as an RC equivalent circuit as below. The resistance and capacitance varies with cable length as follows: \( R = 0.54l \), \( C = 88 \times 10^{-12}l \) where \( l \) is the length of the cable in metres.

![Circuit Diagram]

The sender transmits a logical ONE (1) as 2.4 Volts, and a logical ZERO (0) as 0.4 Volts. The receiver detects a logical ONE received when the received voltage is greater than 2.0 Volts. It detects a logical ZERO when the received voltage is less than 0.4 Volts. The delay in transmission \( \tau \) is the time taken for the receiver to detect a data transition given a worst case bit stream.

What is the length of cable required to ensure that the delay is less than 2ns?
5 Some observations

- Cable lengths affect RC characteristic of line, so affect delay in xmission, so affect bitrate

- To improve bitrate ... make better cable ... hence the different cable standards have whacky specs for shielding etc etc.

- Generally twisted pair has worst characteristics while proper coax (like your cable TV connection) has pretty good characteristics.

- Umm ... except ADSL gives Mbits/sec down a telephone wire ... twisted pair. Erm... how?

- Its to do with modulation and equalisation techniques.

- Equalisation is easy to understand but bloody hard to do: if your cable is messing up your signal; and you know that its behaving like an RC circuit; then to ‘un-mess’ it, you just have to build something to undo the effect of an RC circuit. This is tricky, but possible (see ADSL). Modulation is another thing altogether:

- The bit transmission example we just dealt with assumes bits are sent raw down the channel: baseband transmission.

- Consider this: any cct with active elements has a response that changes with frequency. You can see that with the bit stream example, if the bits are spaced closer together (higher frequency), less gets through, further apart (lower frequency): more gets through.

- In general, there are particular frequencies at which the channel respons better.

- So if we could take our bits, and affect some other signal that is somehow centred at that frequency e.g. modulate the amplitude of a sinusoid (Amplitude Modulation : AM), or modulate the frequency of a sinusoid (Frequency Modulation: FM). Then the bitrate could be increased alot.
• ADSL does this .. with a scheme called QAM (Quadrature Amplitude Modulation). Don’t ask .. its like AM/FM but not really.

• The payback for using modulation\(^1\) is that the modulation/demodulation schemes need more complex circuits to effect.

• These days a really interesting research is being investigated to try to use DIGITAL SIGNAL PROCESSING to do all modulation/demodulation in software. This is a whole lot simpler, but a dammed sight more expensive\(^2\).

• This is NOT the whole story. The other reason that modulation techniques are used is for robustness to NOISE.

• FM radio has better quality sound than AM radio because less noise power is transmitted to the receiver.

• Its the same with ADSL: that whacky modulation scheme has more to do with making sure the bits don’t get drowned in noise.

• To understand these effects you need to know just a little bit about power spectra, and the frequency content of NOISE. But we will stop here in this course.

• Just remember that this simple phenomenon that we just modelled with a dopey RC circuit is at the root of alot of communications scenarios ...eventually leading to the design of mobile phones\(^3\)

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\(^1\)You **never** get something for nothing, no matter what your bank says. This is a *fundamental law of the universe*.

\(^2\)Again: you don’t get something for nothing

\(^3\)Which operate in the most adverse communications environment possible. They would not exist without the work of alot of people designing modulation schemes and integrated circuit implementations.
6 Summary

- Cables can be modelled as RC circuit networks
- The transient response of the network is responsible for the delay in transmission and limits the transmission speed
- A circuit responds differently to different frequencies
- Getting around the problems of transient and frequency response is what efficient communications is about