1. Verify Parseval’s Theorem for the double sided exponential pulse $x(t)$ defined as

$$x(t) = \exp\left(-\frac{|t|}{T}\right)$$

Where $T$ is some constant. You will need

$$\int \frac{1}{(a^2 + x^2)^2} \, dx = \frac{1}{2a^3} \tan^{-1}\left(\frac{x}{a}\right) + \frac{x}{2a^2(a^2 + x^2)}$$
2. The signal $x(t)$ shown below is a periodic waveform consisting of a pulse 0.5 seconds wide repeated every second. The height of the pulse is 1V.

(a) **Without using tables** find an expression for the spectrum of the signal, $X(\omega)$ and calculate the ratio of the amplitude of the 2nd harmonic to the fundamental.

(b) Sketch $|X(\omega)|$ showing important points.

(c) $x(t)$ is input into the $RC$ network as shown below, the resulting output signal is $y(t)$. Find an expression for the spectrum of $y(t)$ after all transients have decayed.

(d) What would you expect to happen to the ratio of the amplitude of the 2nd harmonic to the fundamental in the signal $x(t)$ after it is processed through the system? (You do not need to perform any calculations here.)

![Diagram for Question 2](image)

**Figure 1:** Diagram for Question 2.
3. Given \( x(t) \leftrightarrow X(\omega) \) and \( y(t) \leftrightarrow Y(\omega) \), prove that \( x(t) * y(t) \leftrightarrow Y(\omega)X(\omega) \)

**Without using tables**, find the Fourier Transform of the signal \( x_1(t) \) below. Hence or otherwise, find the Fourier Transform of the signal \( x_2(t) \).

Given that \( y(t) = x_1(t) * x_2(t) \) show that

\[
Y(\omega) = 400[2 - 3\cos(2\omega)]\text{sinc}^2(\omega)
\]

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**Figure 2**: Diagram for Question 3.
4. An electrical system has a transfer function $H(s)$ defined as follows

$$H(s) = -\frac{R_2}{R_1} \times \frac{1 + sC_1R_1}{1 + sC_2R_2}$$

where $R_1 = 5K\Omega$, $R_2 = 10K\Omega$; $C_1 = 1\mu F$; $C_2 = 100\mu F$.

(a) Sketch the Bode Diagram of this system showing clearly the asymptotes you use.

(b) A signal $v_i(t) = A\cos(\omega_0t)$ is input into the system. What is the output signal $v_o(t)$ after all the initial transients have decayed in the following two cases

i. $\omega_0 = 0.5 \text{ rad/sec; } A = 2\text{ Volts.}$

ii. $\omega_0 = 100 \text{ rad/sec; } A = 2\text{ Volts.}$

(c) Does this system exhibit low, high or bandpass filter behaviour?
5. A system is described by the Transfer function

\[ H(s) = \frac{4(s + 2.5)}{s^2 + 2s + 10} \]

(a) Identify the poles and zeros of this system and plot their locations on the s-plane.

(b) From this diagram, sketch the gain of the system as a function of frequency.

(c) Find the 3dB Bandwidth of the system using your sketch. Show clearly on your sketch the range over which this bandwidth is calculated.

6. A linear system has an impulse response

\[ h(t) = \begin{cases} 
3e^{-2(t-5)} & \text{For } t \geq 5 \\
0 & \text{Otherwise}
\end{cases} \]

Find the transfer function of the system. If an input \( x(t) = \sin(\omega t) \) is applied to the system, find the phase lag between the output and input when \( \omega = 1.152 \text{ rad/sec} \) and all initial transients have decayed.
7. (a) Show that the periodic impulse train $s(t)$, having period $T_s$ as indicated in the figure below, can be expressed as

$$s(t) = \frac{1}{T_s} \sum_{k=-\infty}^{\infty} e^{-j\omega_0 kt}$$

where $\omega_0 = \frac{2\pi}{T_s}$.

(b) A signal $x(t)$, having spectrum $X(\omega)$ is sampled to create the signal $x_s(t)$ such that $x_s(t) = x(t) s(t)$, where $s(t)$ is the signal as described above. Derive an expression for $X_s(\omega)$, the spectrum of the sampled signal $x_s(t)$. Given $X(\omega)$ is as shown below, sketch $X_s(\omega)$, using $T_s = 1sec$.

Figure 3: Diagram for Question 7.

Figure 4: Diagram for Question 7(b).
8. A system has a transfer function $H(s)$ defined as follows.

$$H(s) = \frac{3s^2 + 8s + 5}{(s + 2)(s^2 + 2s + 1)}$$

(a) Find the impulse and step responses of the system.

(b) What is the amplitude of the response of the system to a unit step as $t \to \infty$. 