Channel Capacity

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recall

- The last day we looked at some models for discrete memoryless channels
- We looked at the capacity of the binary symmetric channel and did a calculation.
- We now write a general expression for this.

recall from the last day

- When the numbers on the previous slide are inserted we get $H(X|Y) = 0.467$ bits per use of the channel
- Hence $I(X;Y) = H(X) - H(X|Y) =$ **0.504 bits per use of the channel**

what is the maximum value this could be?

- It turns out if you play around with the figures that the maximum capacity you can achieve is when the probability of X and Y is equal, i.e. $p(x)=p(y)=0.5$, just like in the case of the noiseless channel

$$\max(I(X;Y)) \Leftrightarrow P(X=1) = P(X=0) = \frac{1}{2}$$

now lets look again more generally

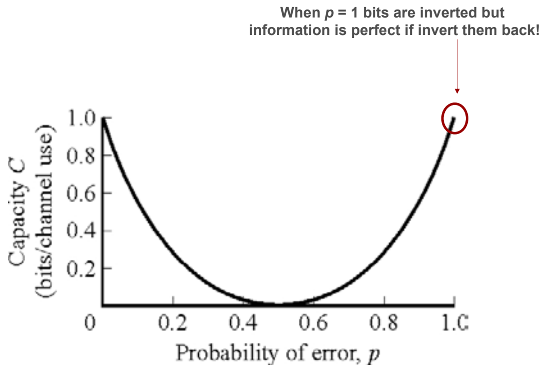
- $I(X;Y) = H(Y) - H(Y|X)$
- $H(Y|X) = -\sum_{x\in\chi} \sum_{y\in Y} p(x,y) \log p(y|x)$
- using the product rule in which $p(x,y) = p(y|x)p(x)$ we can make a substitute above
- $H(Y|X) = -\sum_{x\in\chi} p(x) \sum_{y\in Y} p(y|x) \log p(y|x)$
- $H(Y|X) = -\sum_{x\in\chi} p(x)$ [plogp+(1-p)log(1-p)]

this is 1

So

- $I(X;Y) = H(Y) - [p\log p+(1-p)\log(1-p)]$
- $I(X;Y) = H(Y) - H(p)$
- Recall that we get a maximum when $p(y=0) = p(y=1) = 0.5$
- Hence
- $I(X;Y) = 1 - H(p)$ bits per channel use

maximum channel capacity for BSC



Putting Errors in Context

- The presence of noise in a channel causes errors
- For a relatively noisy channel the probability of error may be around 10^{-2} – this means on average that 99 out of every 100 bits are received correctly
- This is NOT enough
- We need errors of the order of at least 10^{-6}
- Hence we need error coding – the design goal of channel coding is to increase the resistance of a digital communication system to noise

block codes (briefly)

In block coding, we divide our message into blocks, each of k bits, called **datawords**.

We add r redundant bits to each block to make the length n = k + r.

The resulting n bit blocks are called **codewords**.

so

- each k bit block is mapped to an n bit block where n>k
- the number of redundant bits is n- k
- the ratio k/n is called the **code rate r**
- r is less than unity
- the greater the amount of redundancy added the lower the rate

bringing time into it

- If H(X) is the entropy of a source that produces symbols every T_s seconds and the DMC has capacity C and be used once every T_c seconds, then if

$$H(X)/TS \leq C/TC$$

- there exists a coding scheme that can deliver arbitrarily small probability of error – i.e. can reduce the p we spoke about.

we will return to block codes properly in a while

Shannon's theorem:

- A given communication system has a maximum rate of information C known as the **channel capacity**.
- If the information rate R is less than C, then one can approach arbitrarily small error probabilities by using intelligent coding techniques.
- To get lower error probabilities, the encoder has to work on longer blocks of signal data. This entails longer delays and higher computational requirements.

Thus, if $R \leq C$ then transmission may be accomplished without error *in the presence of noise*.

Unfortunately, Shannon's theorem is not a constructive proof — it merely states that such a coding method exists. The proof can therefore not be used to develop a coding method that reaches the channel capacity.

The negation of this theorem is also true: if $R > C$, then errors cannot be avoided regardless of the coding technique used.

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