TEXTURE ANALYSIS: AN ADAPTIVE PROBABILISTIC APPROACH

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ABSTRACT

Two main issues arise when working in the area of texture segmentation: the need to describe the texture accurately by capturing its underlying structure, and the need to perform analyses on the boundaries of textures. Herein, we tackle these problems within a consistent probabilistic framework. Starting from a probability distribution on the space of infinite images, we generate a distribution on arbitrary finite regions by marginalization. For a Gaussian distribution, the computational requirement of diagonalization and the modelling requirement of adaptivity together lead naturally to adaptive wavelet packet models that capture the 'significant amplitude features' in the Fourier domain. Undecimated versions of the wavelet packet transform are used to diagonalize the Gaussian distribution efficiently, albeit approximately. We describe the implementation and application of this approach and present results obtained on several Brodatz texture mosaics.

1. INTRODUCTION

Texture plays an important role in the analysis of images. The need to describe it accurately forms an integral part of many classification, segmentation, and retrieval methods in various application areas [1]. Over the years, many different approaches have been developed to analyse texture, including statistical, geometrical, model and spectral based methods. For a full overview, see [1] and [2].

One research area which has been extremely active in recent years is the application of wavelets to texture analysis [3, 4]. By providing a multiresolution view of the image, wavelets are the perfect tool for examining texture at different scales. An example is the Hidden Markov Tree technique developed by [5], which describes the interscale dependencies of a standard wavelet decomposition. Others have investigated the use of wavelet packets as a texture analysis tool. In [6], packets were used in a classification experiment on natural textures. Since [7], many attempts have been made to adapt the wavelet (or wavelet packet) decomposition to the underlying structure of the texture, for example [8] and [9], but these methods have not been developed within a coherent probabilistic framework.

We address the issue of texture description within a probabilistic framework. Starting from probability distributions for infinite textures, in this paper assumed Gaussian, we derive the distribution for the texture on a finite region. This leads naturally to a class of adaptive wavelet packet models that capture the structure of a given texture, for example its principle periodicities, in a manner analogous to the Wold decomposition [10]. A simple classification rule enables pixelwise classification of the image while retaining the advantages of more complicated prior models.

The paper is organized as follows. Section 2 details the development of our models. In section 3, we describe how we learn the model parameters. Section 4 describes the application of our models to the segmentation of textured images. In section 5, we show results of the segmentation procedure on Brodatz [11] texture mosaics. Finally, in section 6, we conclude and discuss future work.

2. MODELLING PLANAR PARALLEL TEXTURE

One of the characteristics of planar texture, perhaps its defining characteristic, is that it is infinitely extendable, so that images are functions on an infinite (or at least very large) domain D_∞. Thus in order to model such textures accurately, one needs a distribution over the space of such images. We denote such a distribution by:

$$\Pr(I|\mu \equiv m, K_m) \tag{1}$$

where I is the infinite image; K_m is the set of parameters of the model of texture $m \in M$, the set of textures; and $\mu : D_\infty \to M$ is the class map, which here takes every pixel to texture m.

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For practical applications such as image segmentation, one needs to be able to analyse and segment images that contain many different arbitrarily shaped finite textured regions, which means that one needs, not the distribution on infinite images, but that on finite, arbitrarily shaped images. We thus need to marginalize equation (1) over the values of the pixels outside the desired region.

Let Φ be the space of infinite images and $R \subset D_\infty$ a region. There exist two projections, P_R and $P_{\bar{R}}$, and two injections, i_R and $i_{\bar{R}}$, which generate an orthogonal decomposition of Φ:

$$\begin{tikzcd} & \Phi \arrow[dl, shift right, "P_{\bar{R}}"'] \arrow[dr, shift left, "P_R"] & \\ \Phi_{\bar{R}} \arrow[ur, shift right, "i_{\bar{R}}"'] & & \Phi_R \arrow[ul, shift left, "i_R"] \end{tikzcd} \tag{2}$$

$$\Phi \stackrel{\sim}{\leftrightarrow} P_R(\Phi) \otimes P_{\bar{R}}(\Phi) = \Phi_R \otimes \Phi_{\bar{R}} \tag{3}$$

If we now marginalise $\Pr(I|\mu \equiv m, K_m)$ over $\Phi_{\bar{R}}$, we will end up with the probability measure for the image on the finite region R:

$$\Pr(I_R|\cdot) = \int_{\Phi_{\bar{R}}} \Pr(I|\cdot) \tag{4}$$

which, in principle at least, solves the boundary problem for texture.

2.1. Gaussian Distribution

In this paper, as mentioned in section 1, we choose to model texture using a Gaussian distribution, developing this model as outlined above. In abstract notation, a Gaussian distribution can be expressed as:

$$P(I|\cdot) = |F|^{1/2} e^{-\langle I|F|I\rangle} \tag{5}$$

where $\langle I|J\rangle$ is the inner product of the functions $|I\rangle$ and $|J\rangle$ in the space of images, and $|F|$ is the determinant of the operator F. In the position basis, this distribution takes the form:

$$P(I|\cdot) = |F|^{1/2} e^{-\sum_{(x,x')\in D_\infty} I(x)F(x,x')I(x')} \tag{6}$$

where $F(x,x')$, the inverse covariance matrix, captures spatial correlations in the image I. It is diagonal in the position basis only if the values of the pixels in the image are independent of each other.

Corresponding to the orthogonal decomposition of the space of infinite images by the maps P_R and $P_{\bar{R}}$, the operator F can be split up as follows:

$$F = \begin{pmatrix} F_{RR} & F_{R\bar{R}} \\ F_{\bar{R}R} & F_{\bar{R}\bar{R}} \end{pmatrix} \tag{7}$$

where F_{RR} relates the pixels in R to each other, $F_{R\bar{R}}$ relates pixels in $\bar{R}$ to those in R, $F_{\bar{R}R}$ relates pixels in R to those in $\bar{R}$, and $F_{\bar{R}\bar{R}}$ relates pixels in $\bar{R}$ to each other.

Partitioning the operator F in this manner and marginalising over $\Phi_{\bar{R}}$, as in equation (4), gives us the following probability measure for the image I_R on the finite region R:

$$\Pr(I_R|\cdot) = Z^{-1} e^{-\langle I_R|G_R|I_R\rangle} \tag{8}$$

where $G_R \equiv F_{RR} - F_{R\bar{R}}\,(F_{\bar{R}\bar{R}})^{-1}\,F_{\bar{R}R}$.

Although in principle we can evaluate this operator and the exponent of equation (8), in practice computational complexity requires that we diagonalize G_R.

2.2. Diagonalization of the operator G_R

If we can find a set $B = \{|a\rangle \in \Phi_R : a \in A\}$ of functions on the region R such that:

1. The set $\{i_R|a\rangle : a \in A\}$ of infinite images are eigenfunctions of the operator F (with eigenvalues f_a);

2. The set B forms an orthonormal basis for Φ_R;

then we can diagonalize the operator G_R. The first condition implies that the support of $F i_R|a\rangle$ lies in the region R. Thus the second term in G_R is zero, and the first term becomes $f_a|a\rangle$. Hence:

$$\langle a|G_R|a\rangle = f_a\langle a|a\rangle \tag{9}$$

The second condition then means that G_R is diagonalized by B, allowing us to write our distribution as:

$$\Pr(I_R|\cdot) = Z^{-1} e^{-\sum_{a\in A} f_a\langle I_R|a\rangle\langle a|I_R\rangle} \tag{10}$$

How do we find such a set B?

2.3. Using Wavelet Packets

One of the characteristics we demand from our distribution is that of translational invariance. This condition makes our operator F diagonal in the Fourier basis, $F(k,k') = f(k)\delta(k,k')$, which means that our distribution is now characterized by a function f on the Fourier domain:

$$\Pr(I|\cdot) = |F|^{1/2} e^{-\sum_k f(k)I^*(k)I(k)} \tag{11}$$

For an arbitrary function $f(k)$, it is very hard to find a set B that satisfies the conditions in Section 2.2. We thus want to choose a set of functions f that is varied enough to capture the structure present in the texture, but limited enough that we can satisfy the conditions. To this end, consider the set $\mathcal{T}$ of dyadic partitions of the Fourier domain. We define a set of functions $\mathcal{F}$ by:

$$\mathcal{F} = \bigcup_{T\in\mathcal{T}} \mathcal{F}_T \tag{12}$$

where $\mathcal{F}_T = \{f : f \text{ is piecewise constant on } T\}$. Given an element of $T \in \mathcal{T}$, and a mother wavelet, we can define a wavelet packet basis B_T. Each element of this basis has frequency support that lies approximately in one of the elements of the partition T. The basis elements are thus approximate eigenfunctions of the operators defined by the functions in $\mathcal{F}_T$. Those basis elements whose support lies in R thereby satisfy condition 1.

Our next task is to complete the set of wavelets inside the region R in order to make a basis for the region and in doing so satisfy condition 2. How we do this depends on the shape of R. We consider two possibilities: dyadic and arbitrarily shaped regions.

3. PARAMETER ESTIMATION

For the first case, we can use a decimated wavelet packet decomposition to obtain a basis for R. Given a partition T and a function $f \in \mathcal{F}_T$, the distribution for a dyadic region R takes on the form:

$$\Pr(I_R|f,T) = \prod_\alpha \left[\left(\frac{f_\alpha}{\pi}\right)^{N_\alpha/2} e^{-f_\alpha \sum_{i\in\alpha} w_{\alpha,i}^2} \right] \quad (13)$$

where α is the index for the subbands of T; f_α is the value of f on subband α; i is an index for the individual wavelets within each subband; $w_{\alpha,i}$ is the $\langle \alpha, i\rangle$ wavelet coefficient of the image; and N_α is the number of coefficients in subband α.

When estimating the parameters of a texture, we can choose to use dyadic sample images. In our experiments, for each texture, we used 64 patches, each of size 256×256. To find the optimal parameters for a given texture we examine the probability

$$\Pr(f,T|d) \propto \Pr(d|f,T)\Pr(f|T)\Pr(T) \quad (14)$$

where d is the training data used for a given texture.

We assume $\Pr(f|T)$ to be uniform, and choose the prior, $\Pr(T)$, to penalize large decompositions:

$$\Pr(T) = Z^{-1}(\beta)e^{-\beta|T|} \quad (15)$$

where $|T|$ is the number of elements in the partition. The probability $\Pr(d|f,T)$ is given by equation (13). Differentiating with respect to f_α gives us the maximum a posteriori (MAP) estimate of f for fixed T:

$$\hat{f}_\alpha = \frac{N_\alpha}{2\sum_{i\in\alpha} w_{\alpha,i}^2} \quad (16)$$

We use a depth-first search through the space $\mathcal{T}$ to find the exact MAP estimates for both T and f. Figure 1 shows two examples of texture models that were trained using this algorithm.

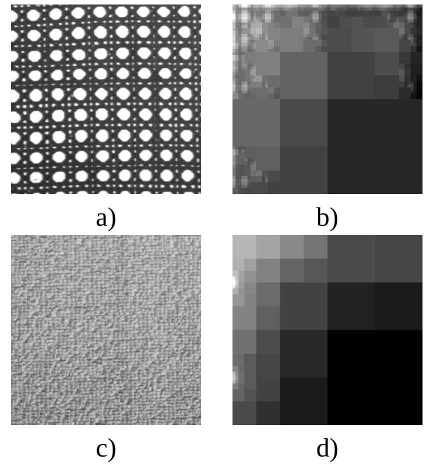

Fig. 1. a) Texture D101 and b) its optimal decomposition; c) Texture Raffia and d) its optimal decomposition.

4. CLASSIFICATION

For arbitrarily shaped regions R, dyadic wavelet packets no longer form a basis. There are two problems. First, the basis elements may not be aligned with the boundary and so include information from outside R. Second, a shifting of R with respect to the basis elements will produce a different representation of the same texture.

To ameliorate this situation, we complete the basis in R using the following approximate scheme. For each subband, we take the geometric mean, over all translations, of the probabilities of the parts of the translated versions of the subband that lie within R, and then recombine these probabilities to give a probability for the image in R. The effect is to create an undecimated wavelet decomposition of R. The distribution takes on the following form:

$$\Pr(I_R|f,T) = \prod_\alpha \prod_{x\in R} \left[\left(\frac{f_\alpha}{\pi}\right)^{\frac{1}{2M_\alpha}} e^{-\frac{f_\alpha}{M_\alpha} w_{\alpha,x}^2} \right] \quad (17)$$

where M_α is the redundancy factor for subband α: the number of pixels between wavelets in the subband. Note that this distribution is not the same as that found by assuming that the coefficients in the undecimated wavelet decomposition are independently distributed.

Given the probability of a region of each texture, we assume that the probability of a finite composite image I_D, with domain D and class map $\mu : D \to M$, is:

$$\Pr(I_D|\mu, \{K_m\}) = \prod_{m\in M} \Pr(I_{R_m}|\mu_{R_m} \equiv m, K_m) \quad (18)$$

where $R_m \subset D = \mu^{-1}(m)$, the region with class m. The probability of a class map is then given by:

$$\Pr(\mu|I_D, \{K_m\}) \propto \Pr(I_D|\mu, \{K_m\})\Pr(\mu) \quad (19)$$

With a trivial prior, due to the form of equation (17), we could perform a pixelwise maximum likelihood classification of the image. In practice, we know that μ is likely to be somewhat regular. We could define a Potts prior and use simulated annealing to make a MAP estimate of μ. This of course is very slow, and it turns out that another approach produces results that are as good, if not better, while remaining extremely quick. We use the following classification rule:

$$\hat{\mu}(x) = \arg\max_{m \in M} \prod_{x' \in V(x)} \Pr(I_D(x)|\mu(x') = m) \qquad (20)$$

where $V(x)$ is the set of neighbours of pixel x. This rule has a similar effect to the Potts prior, but it still allows a pixelwise classification because it uses the data at the neighbours of a pixel but not their unknown classes. In consequence, one can use larger neighbourhoods with little extra penalty.

5. RESULTS

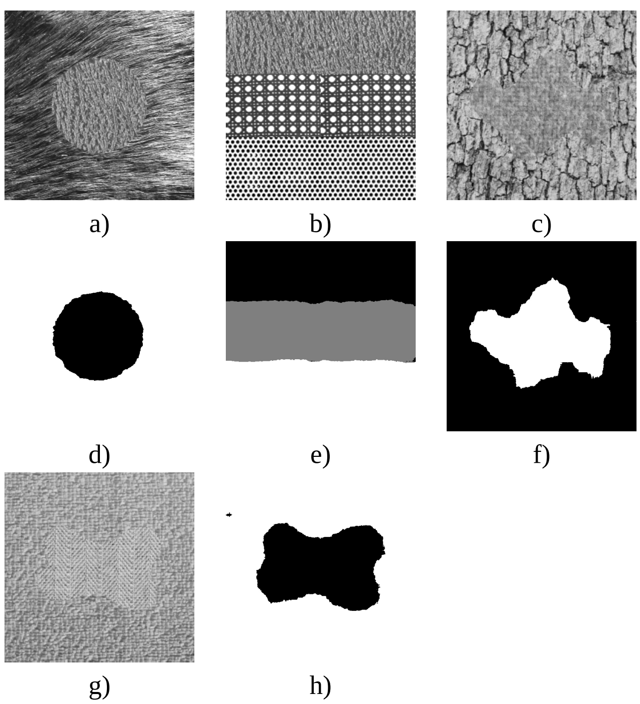

Fig. 2. a) Circular mosaic of Calf and Fabric0004 and d), its segmentation; b) Rectangular mosaic of Calf, D101, and Hexholes152 and e), its segmentation; c) Freehand mosaic of Bark and Wool and f), its segmentation; g) Freehand mosaic of Herring and Raffia and h) its segmentation.

Figure 2 shows segmentation results on four 512×512 Brodatz texture mosaics. The misclassification percentages were 2.3%, 5.5%, 2.4%, and 2.5% respectively.

6. CONCLUSIONS

We have described a new adaptive probabilistic model for texture description and segmentation. Wavelet packet bases, which arise naturally within our probabilistic framework, allow the model to adapt to an individual texture and in doing so capture its underlying structure. Our model was tested on several Brodatz texture mosaics.

We are currently applying these models, and others developed within the same framework, to remote sensing applications such as detection and verification of land usage and retrieval applications where the database contains highly textured images. Results will be reported at a later date.

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