Paper 3C1

Answers for Examples Sheet 5: Frequency Response

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1. The frequency response for this system is given by (replacing s by $\jmath\omega$ in the transfer function)

$$\begin{aligned} H(\jmath\omega) &= \frac{1}{(\jmath\omega+1)(\jmath\omega+0.1)} \\ &= \frac{1}{(.1-\omega^2)+1.1\jmath\omega} \end{aligned}$$

We are told that, at frequency ω the output phase lags by $\pi/2$. Therefore we have that

$$\begin{aligned} -\pi/2 &= \arg\frac{1}{(.1-\omega^2)+1.1\jmath\omega} \\ &= -\tan^{-1}\left(\frac{1.1\omega}{0.1-\omega^2}\right) \end{aligned}$$

Therefore

$$\begin{aligned} \tan\pi/2 &= \left(\frac{1.1\omega}{0.1-\omega^2}\right) \\ \infty &= \left(\frac{1.1\omega}{0.1-\omega^2}\right) \end{aligned}$$

yielding

$$0.1-\omega^2=0$$

or

$$\omega=\sqrt{0.1}=\frac{1}{\sqrt{10}}$$

The gain at this frequency is given by

$$\begin{aligned} |H(\jmath\omega)|_{\omega=1/\sqrt{10}} &= \left|\frac{1}{(0.1-\omega^2)+1.1\jmath\omega}\right|_{\omega=1/\sqrt{10}} \\ &= \left|\frac{1}{1.1\jmath\frac{1}{\sqrt{10}}}\right| \\ &= \frac{\sqrt{10}}{1.1} \end{aligned}$$

The output signal thus has amplitude

$$Y=\frac{\sqrt{10}X}{1.1}$$

2. The transfer function of the system with $G_1(s)$ and $G_3(s)$ in cascade, is $G_1(s)\cdot G_3(s)$. The Bode diagram of $G_3(s)$ is known. Hence, to simplify, we can first draw the Bode diagram of $G_1(s)$.

Then, the Bode diagram of $G_1(s) \cdot G_3(s)$ is computed by simple addition for both the phase and the gain. We have:

$$\left| \begin{array}{l} 20\log|G_1(s)| = 10\log_{10}\left(\frac{1+\omega^2T^2}{1+a^2\omega^2T^2}\right) \\ \\ \arg[G_1(s)] = \tan^{-1}(T\omega) - \tan^{-1}(aT\omega) \end{array} \right. \quad (1)$$

Using equations 1, the values of the gain and the phase can be computed (cf. table 1). Figures

ω	0	10^{-1}	1	3	6	10	100	inf
$20\log\|G_1(s)\|$	0							$-20\log a$
(a) $a = 0.1$ $T = 1$	0	0	3	9	14	17	19.9	20
(b) $a = 4$ $T = 2.5$	0	-2.7470	-11.4	-12	-12	-12	-12	-12
$\arg[G_1(s)]$	0							0
(a) $a = 0.1$ $T = 1$	0	5	39	55	49.6	39	5	0
(b) $a = 4$ $T = 2.5$	0	-31	-16	-6	-2.8	-1.7	-0.2	0

Table 1: Values to draw the Bode diagram of $G_1(s)$.

1 and 3 represent the Bode diagrams of G_1 for the cases (a) and (b) respectively. In reading the values of the gain and phase of G_3 and adding them to the ones of G_1, the Bode diagram of the system $G_1 \cdot G_3$ can easily be computed (cf. figures 2 and 4).

3. An attenuation of $\frac{1}{\sqrt{2}}$ for the gain corresponds to an attenuation of $-3dB$ on the Bode diagram. In reading the Bode diagram of H_1 (cf. figure 5), the gain is superior to $-3dB$ when $\omega \in [0; 22000]$. H_1 is a low pass filter. In reading the Bode diagram of H_2 (cf. figure 6), the gain is superior to $-3dB$ when $\omega \in [300; 22000]$. H_2 is a band pass filter.

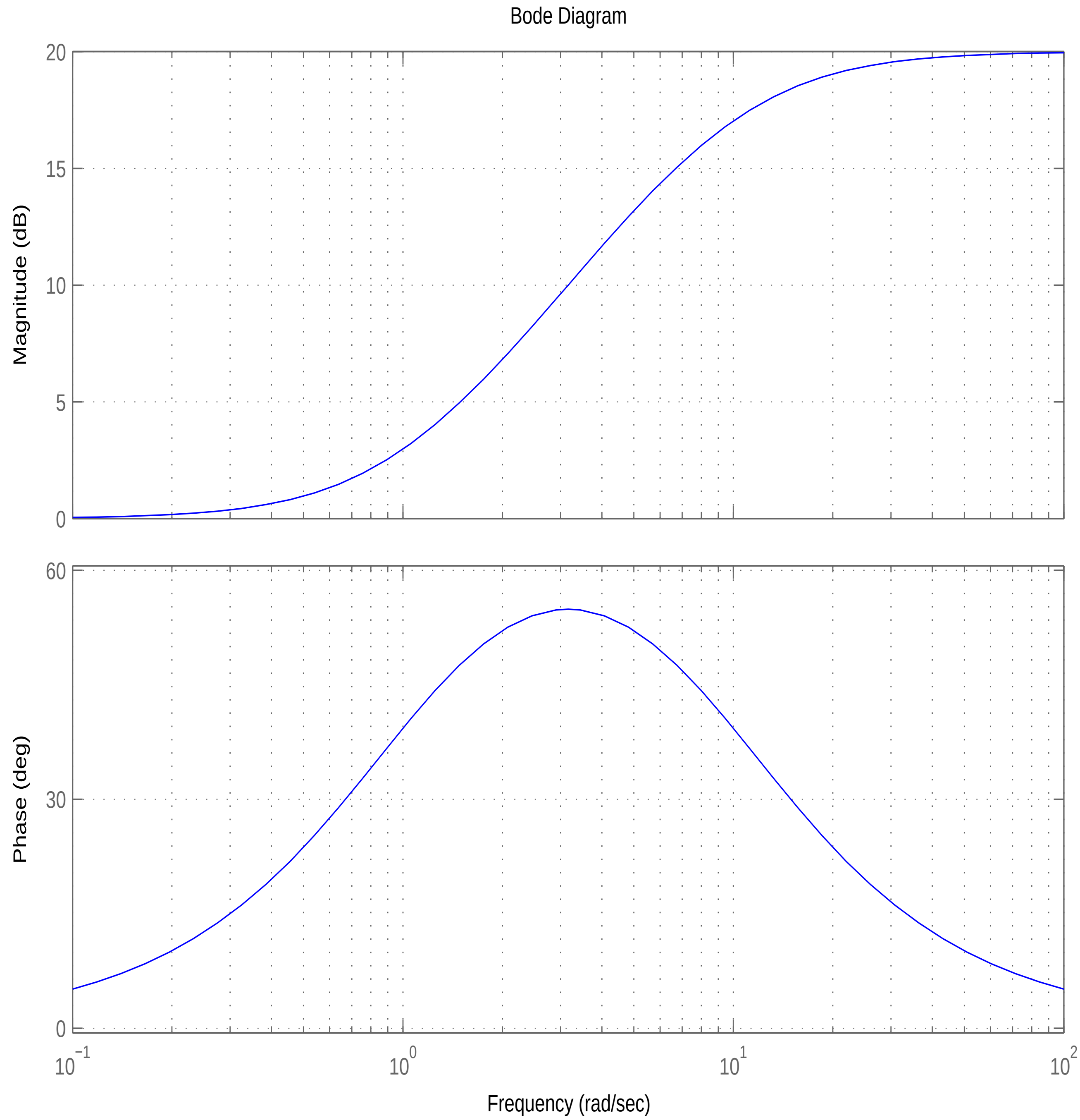

Figure 1: Bode diagram of $G_1(s)$: (a) $a = 0.1$ and $T = 1$.

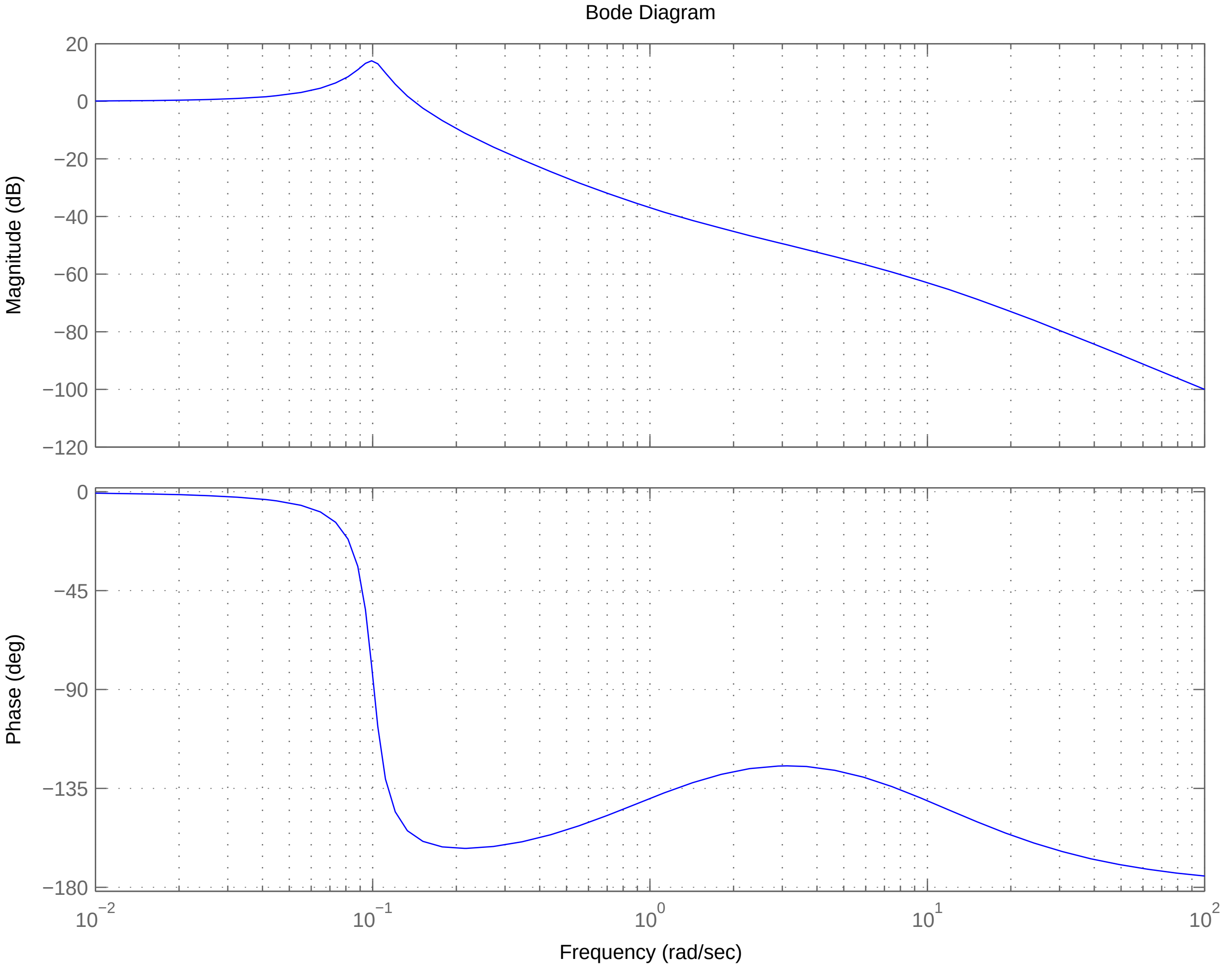

Figure 2: Bode diagram (a) $G_1(s)\ G_3(s)$.

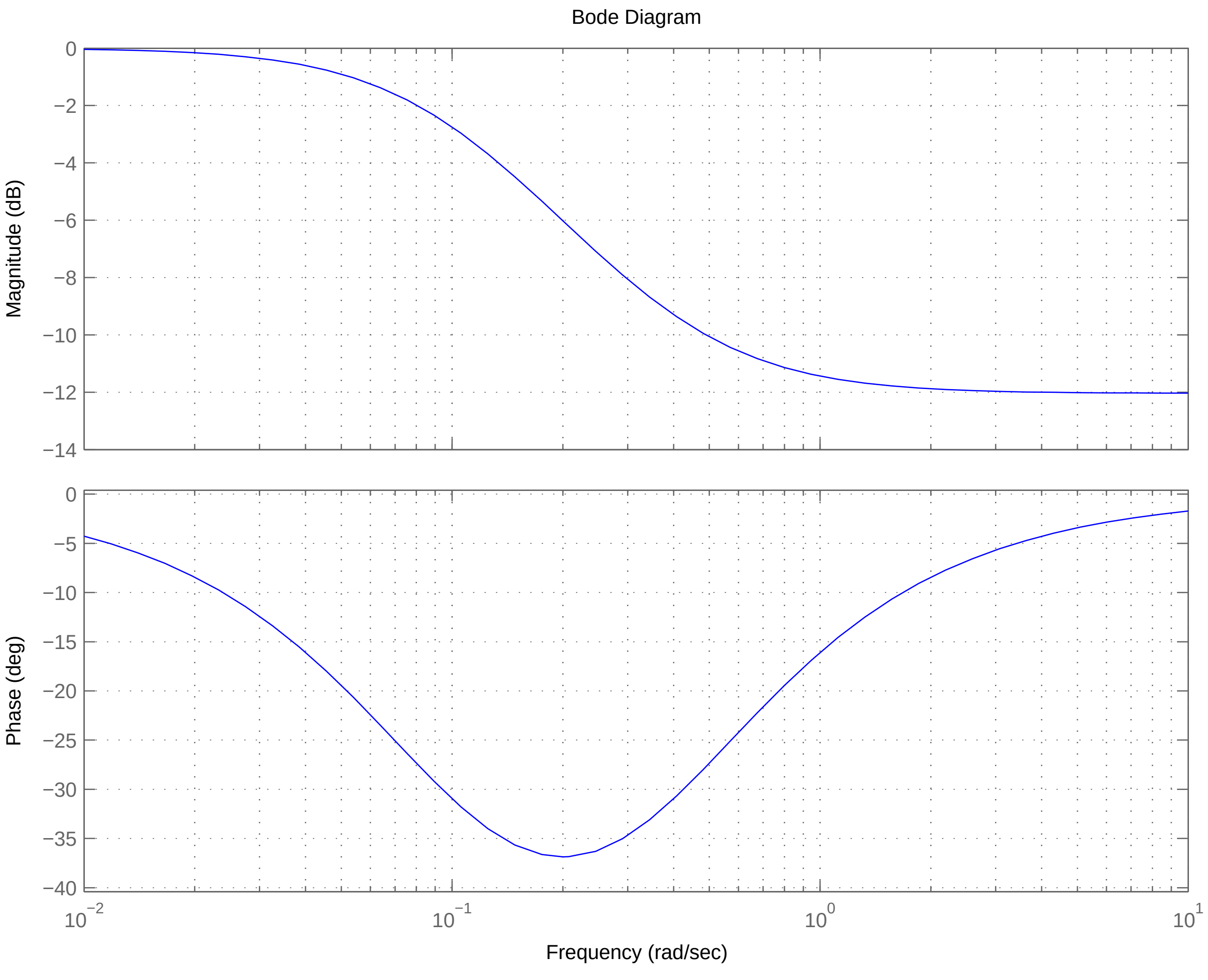

Figure 3: Bode diagram of $G_1(s)$: (b)$a = 4$ and $T = 2.5$.

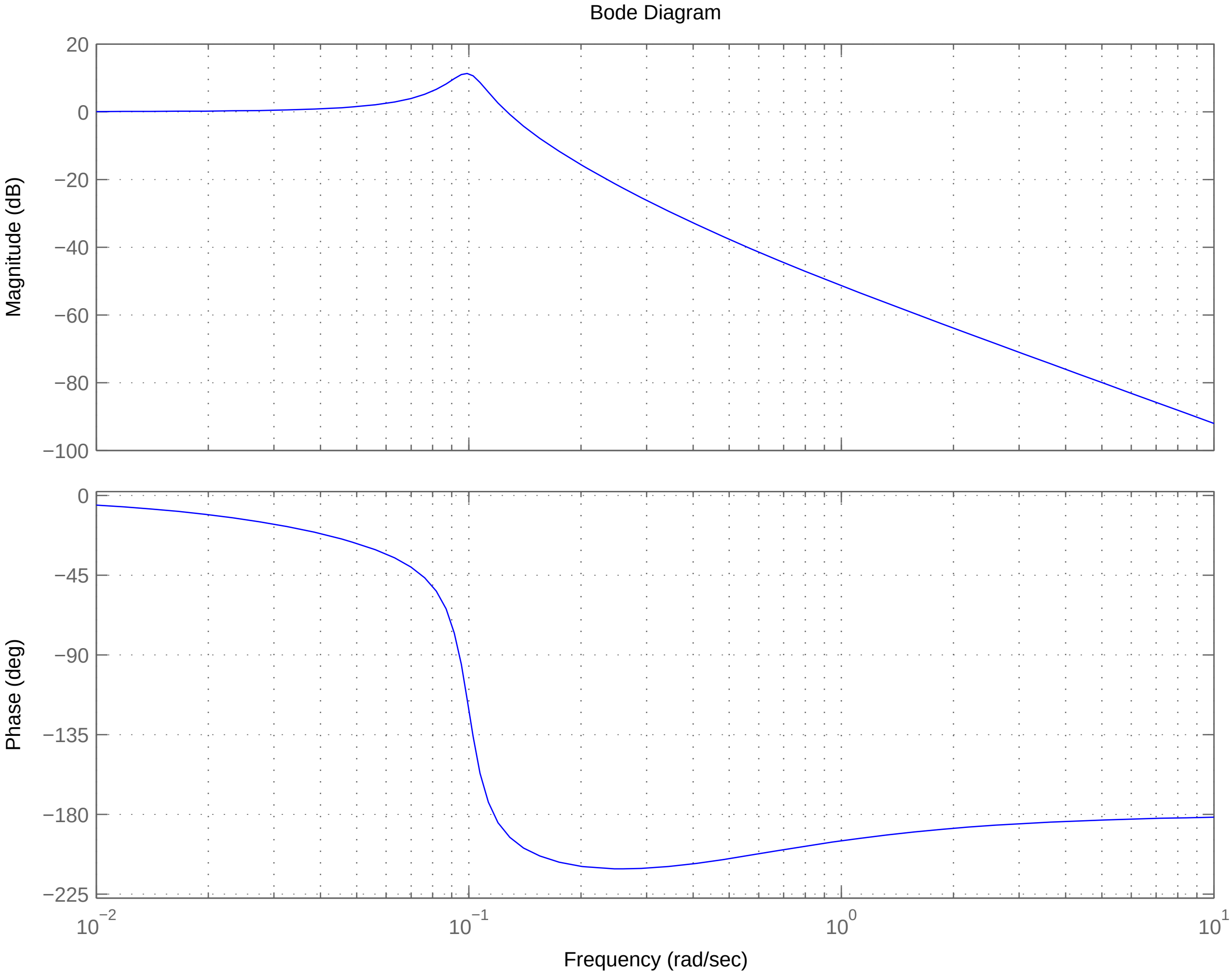

Figure 4: Bode diagram (b) $G_1(s)$ $G_3(s)$.

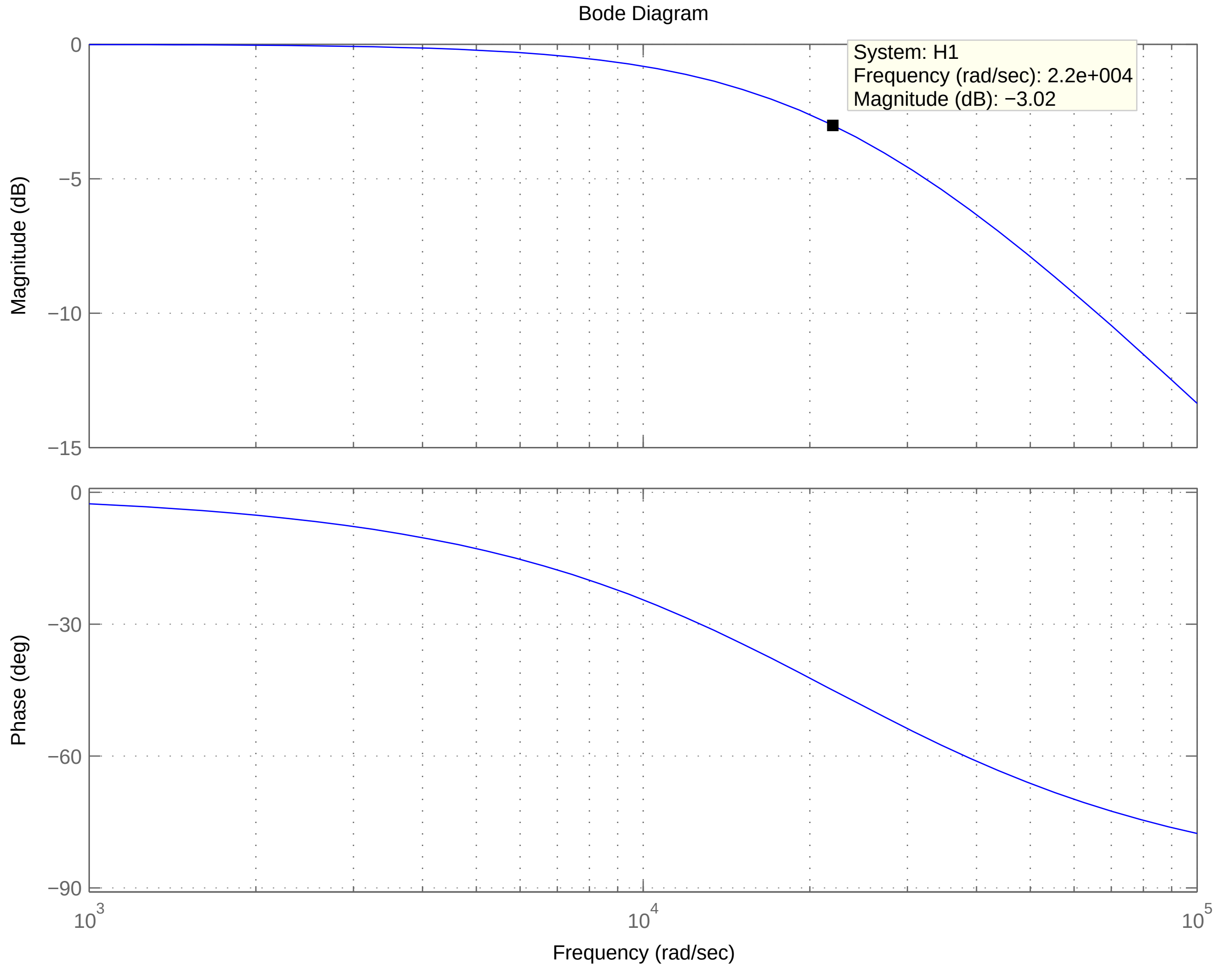

Figure 5: Bode diagram H_1.

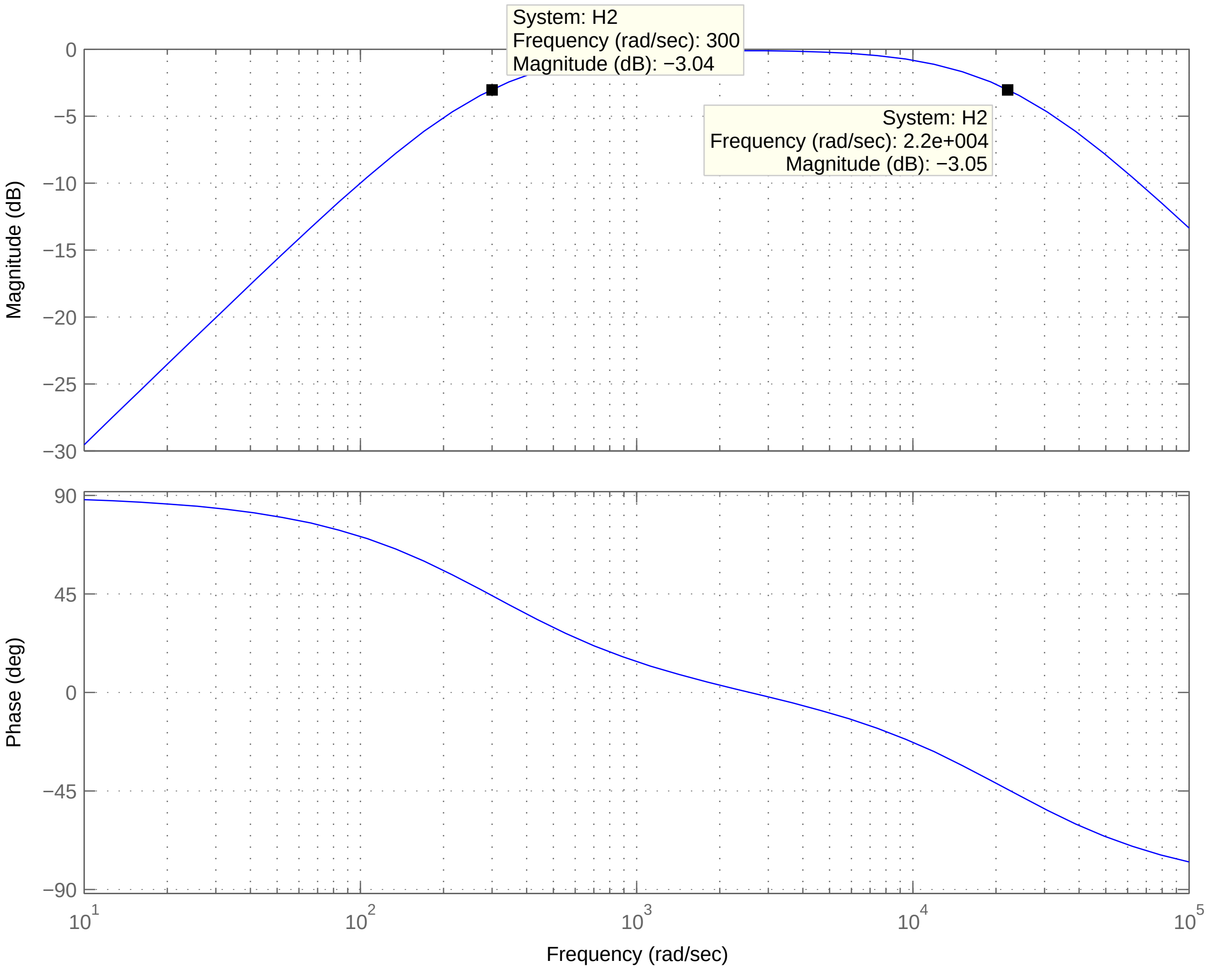

Figure 6: Bode diagram H_2.