

Paper 3C1  
**Examples Sheet 1: Complex numbers**

- Express each of the following complex numbers in polar form ( $re^{j\theta}$ ), (with  $-2\pi \leq \theta \leq 2\pi$ ):  
 $5$ ,  $-3j$ ,  $(1-j)^2$ ,  $\frac{\sqrt{2}+j\sqrt{2}}{1+j\sqrt{3}}$ .
- Express each of the following complex numbers in cartesian form ( $x + jy$ ):  
 $z_1 = \frac{1}{2} \exp(j\pi)$ ,  $z_2 = \exp(j\frac{5\pi}{2})$ ,  $z_3 = \sqrt{2} \exp(-j\frac{\pi}{4})$ .
- Reduce the following complex numbers to the form  $a + jb$  where  $a$ ,  $b$  are real constants:  
 $z_4 = \frac{5}{2+j}$ ,  $z_5 = \frac{z_4+z_2}{z_3+z_2}$ . Show  $z_5 - z_4 = \exp(-j\frac{\pi}{2})(z_3 - z_4)$ : what is the transformation linking  $z_5$  and  $z_3$  (translation or rotation)? Specify its parameters (centre and angle).
- Locate on a graph the complex numbers  $z_1, z_2, z_3, z_4, z_5$  (defined in the previous questions) and their conjugate  $z_1^*, z_2^*, z_3^*, z_4^*, z_5^*$ . What is the geometric relation between a complex number and its conjugate?
- Plot the solutions for  $z$  that satisfy the relation  $|z - z_1| = |z - z_2|$ . (Hint: use geometry).
- Sketch carefully portions of the following exponential and sinusoidal signals (over two periods when the signal is periodic ).
  - $x(t) = \exp(t - 1)$  with  $t \in [-1; +5]$
  - $x_n = 2 \exp(-n/2)$  with  $-1 \leq n \leq 5$
  - $x(t) = A \sin(100\pi t + \frac{\pi}{2})$
  - $x_n = \cos(\frac{\pi n}{6})$
  - $x(t) = \exp(-t) \sin(\frac{\pi t}{2})$  with  $t \in [0; +8]$

Answers

- $5 = 5 \times e^{j2\pi}$ ,  $-3j = 3 \times e^{-j\frac{\pi}{2}}$ ,  $(1-j)^2 = 2 e^{(-j\frac{\pi}{2})}$ ,  $\frac{\sqrt{2}+j\sqrt{2}}{1+j\sqrt{3}} = e^{(-j\frac{\pi}{12})}$
- $z_1 = -0.5 + j 0$ ,  $z_2 = 0 + j$ ,  $z_3 = 1 - j$
- $z_4 = 2 - j$ ,  $z_5 = 2$ .