Paper 3C1

Examples Sheet 3: Laplace Transform

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Laplace Transforms

1. Find the Laplace transforms of the following functions given that these functions are zero for $t < 0$.
   (a) $f(t) = e^{-\alpha t}\cos(\beta t + \phi)$ where α, β are real and positive and ϕ is real
   (b) $f(t) = t^2 + 1$
2. Find the inverse transform of
   (a) $\frac{6}{(s+1)(s+2)(s+3)}$
   (b) $\frac{2}{(s+1)(s^2+2^2)}$
   (c) $\frac{3}{(s+1)(s+2)^2}$

Transfer functions

3. Identify the poles and zeros of the following transfer functions and find the impulse response $h(t)$.
   (a) $\frac{36(s+4)}{s(s+2)(s+8)}$
   (b) $\frac{30(s+1)(s+2)}{s(s+3)(s^2+9s+20)}$
   (c) $\frac{200(s+10)}{s(s+5)^2}$
   (d) $\frac{10(s^2+9s+18)}{s(s+5)}$
4. Find the transfer function of the circuit shown in the figure 1. Assume the amplifier is ideal.

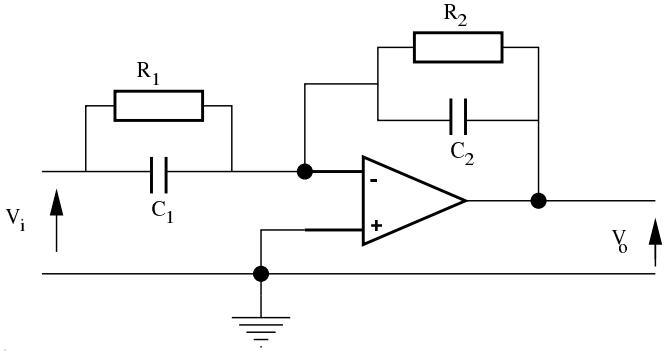

Figure 1: Circuit for question 4.

5. Find the transfer function relating y to x for each of the block diagrams in the figure 2.

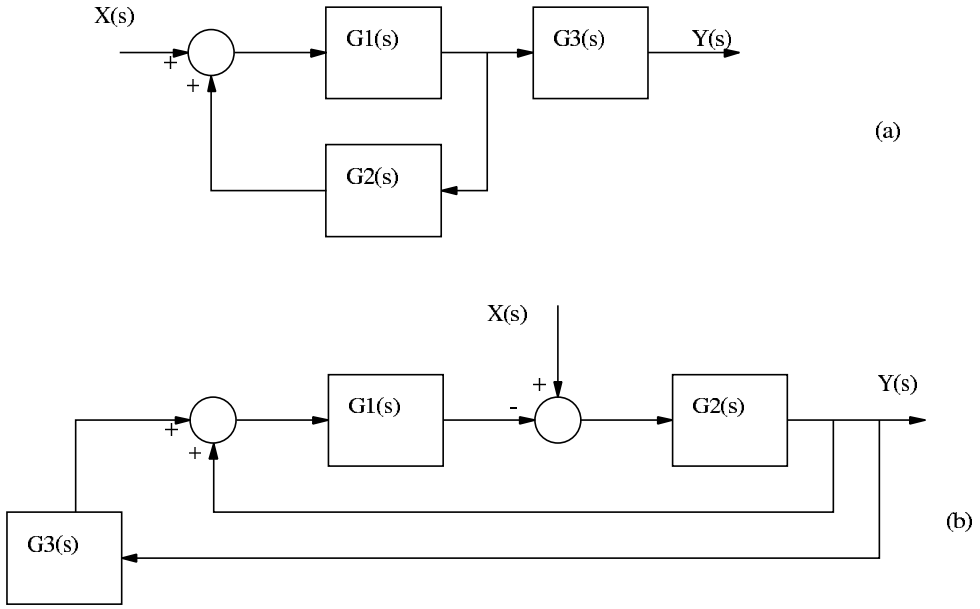

Figure 2: Block diagrams for question 5.

Impulse and Step responses

6. Find expression for the transfer function V_0/I_{in} for the circuit shown fig. 3 and thereby find the unit step and unit impulse responses.

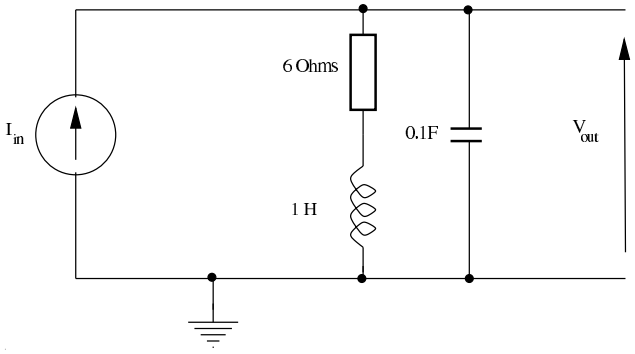

Figure 3: Circuit for question 6.

7. Find an expression for $i(t)$ in the circuit below (cf. fig. 4) for $t > 0$, if the 40 Volt source has been connected for a long time and the switch is moved to position 2 at $t = 0$.

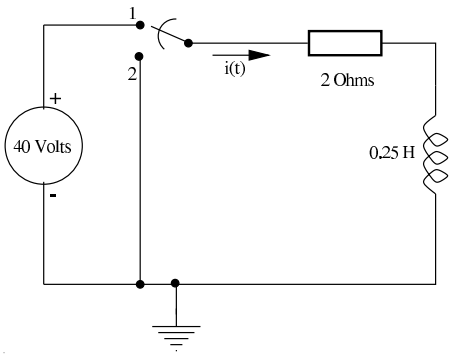

Figure 4: Circuit for question 7.

8. Calculate the impulse response for the system whose behaviour can be modelled by the differential equation

$$\frac{\partial^2 y}{\partial t^2} + 2\alpha\frac{\partial y}{\partial t} + (\Omega^2 + \beta^2)y = x$$

for the following parameters

(a) $\alpha > 0$; $\beta = \alpha$; $\Omega^2 > 0$;

(b) $\beta = \alpha$; $\Omega^2 = -A^2 < 0$;

(c) $\Omega^2 + \beta^2 = \alpha^2$

(d) $\alpha < 0$; $\beta = \alpha$; $\Omega^2 > 0$;

Sketch the impulse response in each case, including scales on both axes.

Answers

1. (a) $\bar{f}(s) = \frac{(s+\alpha)\cos(\phi) - \beta\sin(\phi)}{(s+\alpha)^2+\beta^2}$

   (b) $\bar{f}(s) = \frac{s^2+2}{s^3}$

2. (a) $3e^{-t} - 6e^{-2t} + 3e^{-3t}$

   (b) $\frac{1}{5}\{2e^{-t} - 2\cos(2t) + \sin(2t)\}$

   (c) $3\{e^{-t} - te^{-2t} - e^{-2t}\}$

3. (a) $9 - 6e^{-2t} - 3e^{-8t}$

   (b) $1 - 10e^{-3t} + 45e^{-4t} - 36e^{-5t}$

   (c) $80 - 200te^{-5t} - 80e^{-5t}$

   (d) $10\delta(t) + 36 + 4e^{-5t}$

4. $-\frac{R_2(1+sC_1R_1)}{R_1(1+sC_2R_2)}$

5. (a) $\frac{g_3(s)g_1(s)}{1-g_1(s)g_2(s)}$

(b) $\frac{g_2(s)}{1+g_2(s)g_1(s)[1+g_3(s)]}$

6. $V_{\rm O}/I_{\rm in} = \frac{10(s+6)}{s^2+6s+10}$, Step response $= 6 + 10e^{-3t}\cos(t + 126.87^\circ)$ Volts, Impulse response $= 31.62e^{-3t}\cos(t - 71.57^\circ)$ Volts. **OR (using radians)** Step response $= 6 + 10e^{-3t}\cos(t + 2.214)$ Volts, Impulse response $= 31.62e^{-3t}\cos(t - 1.249)$ Volts.

7. $i(t) = 20e^{-8t}$

8. (a) $\frac{1}{\Omega}e^{-\alpha t}\sin(\Omega t)$
   (b) $\frac{1}{2A}\{e^{-(\alpha-A)t} - e^{-(\alpha+A)t}\}$
   (c) $te^{-\alpha t}$
   (d) $\frac{1}{\Omega}e^{|\alpha|t}\sin(\Omega t)$