

Examples Sheet 4: Convolution and Pole Locations

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Convolution

- Obtain the convolution of the signals $x_1(t)$ with $x_2(t)$ defined below.

$$x_1(t) = \begin{cases} \exp(-at) & t \geq 0 \\ 0 & t < 0 \end{cases}$$

$$x_2(t) = \begin{cases} \exp(-bt) & t \geq 0 \\ 0 & t < 0 \end{cases}$$

Consider the cases $a = b$ and $a \neq b$.

- The path connecting a sender to a receiver in a telephone network can be modelled very approximately as a linear system with impulse response $\beta e^{-\beta t}$ ($t \geq 0$). If the signal at the sending end of the network is $e^{-\alpha t}$, find the received signal by using the convolution theorem.

Repeat the calculation using Laplace Transforms and the transfer function of the network.

Pole Locations

- Classify the following transfer functions as asymptotically stable, marginally stable or unstable:

(a) $H(s) = 1/(s + 3)$

(b) $H(s) = (s - 1)/(s + 3)$

(c) $H(s) = 1/(s - 3)$

(d) $H(s) = 1/s$

(e) $H(s) = 1/s^2$

(f) $H(s) = (2s + 3)/(s^2 + 8)$

(g) $H(s) = (2s + 3)/(s^2 - 8)$ (h) $H(s) = 10/(s + 5)(s^2 - s + 1)$ (i) $H(s) = \frac{10e^{-2s}}{(s + 5)(s^2 + s + 1)}$

In each case draw the pole-zero diagram.

- Figure 1 shows, in the left hand column, the pole locations of some second order systems. The right hand column shows the impulse responses of these systems, but *not in the same order*. Match the diagrams in the left-hand column with the corresponding diagrams in the right-hand column. (Note that some of the systems may indeed have zeros but these are not shown.)

Answers

- For $a = b$, te^{-at} ; for $a \neq b$, $\frac{e^{-bt} - e^{-at}}{a - b}$
- $\beta(e^{-\alpha t} - e^{-\beta t})/(\beta - \alpha)$ if $\beta \neq \alpha$
 $\beta te^{-\beta t}$ if $\beta = \alpha$

3. Asymptotically Stable: (a), (b), (i).

Marginally Stable: (d), (f).

Unstable: (c), (e), (g), (h).

4. $a \leftrightarrow 2$, $b \leftrightarrow 3$, $c \leftrightarrow 1$, $d \leftrightarrow 5$, $e \leftrightarrow 4$.

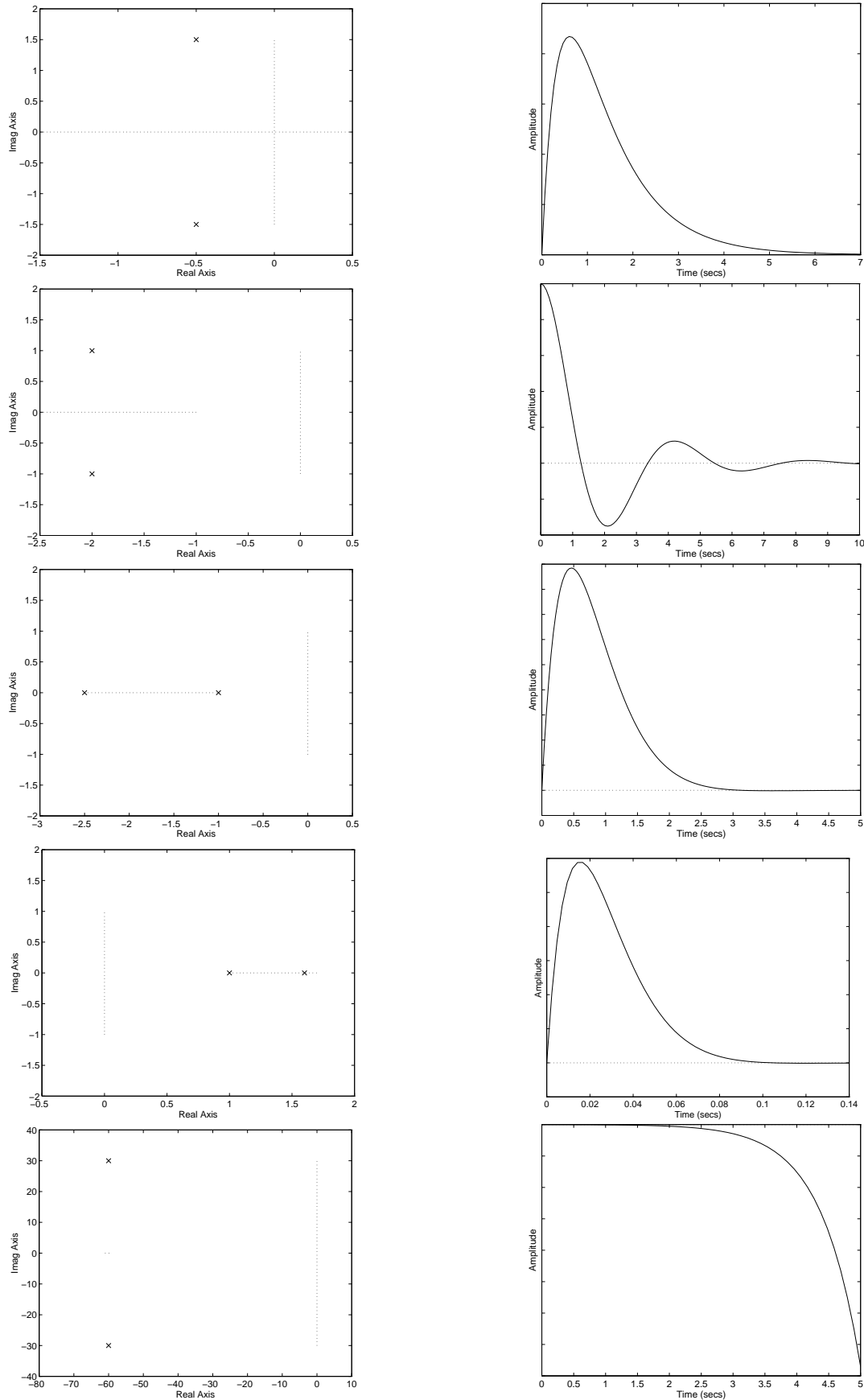


Figure 1: Pole positions and impulse responses.