

## Paper 3C1

### Examples Sheet 5: Frequency Response

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<http://www.mee.tcd.ie/~sigmedia>

1. A sinusoidal input is applied to a system with transfer function

$$H(s) = \frac{1}{(s+1)(s+0.1)}$$

At which frequency does the output lag behind the input by  $90^\circ$ ? if the input amplitude is  $X$ , what is the output amplitude at this frequency?

2. Given the following transfer functions:

$$G_1(s) = \frac{1+sT}{1+asT} ; \quad G_3(s) = \frac{1}{1+2c_3sT_3+s^2T_3^2} \text{ For } c_3 = 0.1, T_3 = 10.$$

and the Bode diagram of  $G_3$  in figure 1, draw on the figure 2 the Bode diagram of the transfer function of  $G_1$  and  $G_3$  in cascade with (a)  $a = 0.1, T = 1$  and (b)  $a = 4, T = 2.5$  (drawing the effects of asymptotes only will suffice).

3. In the last question of Examples Paper 2, the transmission path through a telephone network was modelled as a system with impulse response  $\beta e^{-\beta t}$  and transfer function (taking  $\beta = 22000$ ):

$$\begin{aligned} H_1(s) &= \mathcal{L}\{\beta e^{-\beta t}\} \\ &= \frac{\beta}{s+\beta} \\ &= \frac{22000}{s+22000} \end{aligned}$$

A better model is given by the transfer function:

$$H_2(s) = \frac{22000s}{(s+300)(s+22000)}$$

The Bode diagrams for the two models are given figures 3 and 4.

A sinusoidal signal of frequency  $\omega$  rad/sec is transmitted through the network. Using the Bode diagrams, estimate the range of values of  $\omega$  for which the received signal would be attenuated by less than  $1/\sqrt{2}$ , according to each of the two models. What kind of filter does each system represent?

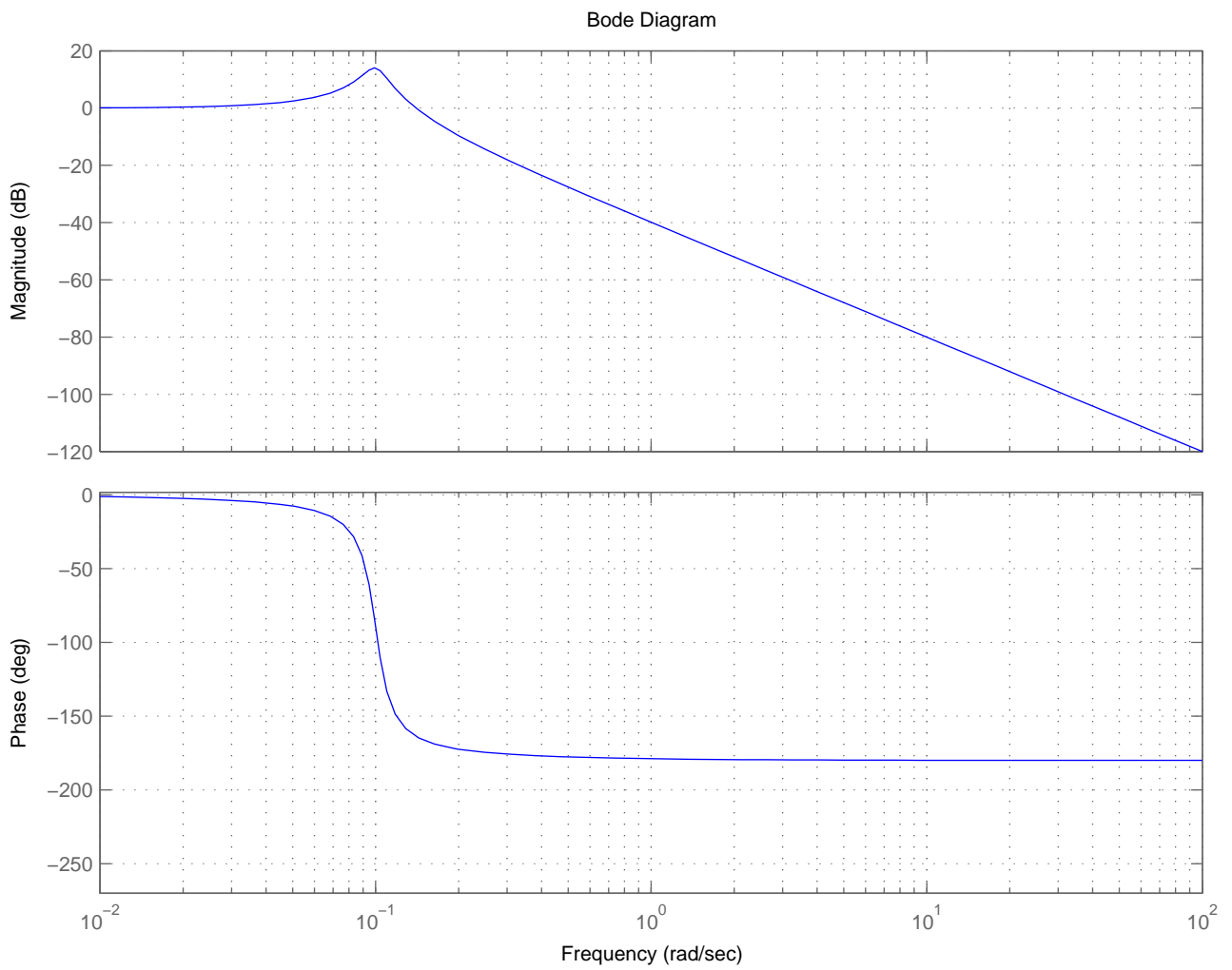


Figure 1: Bode plot for  $G_3$ .

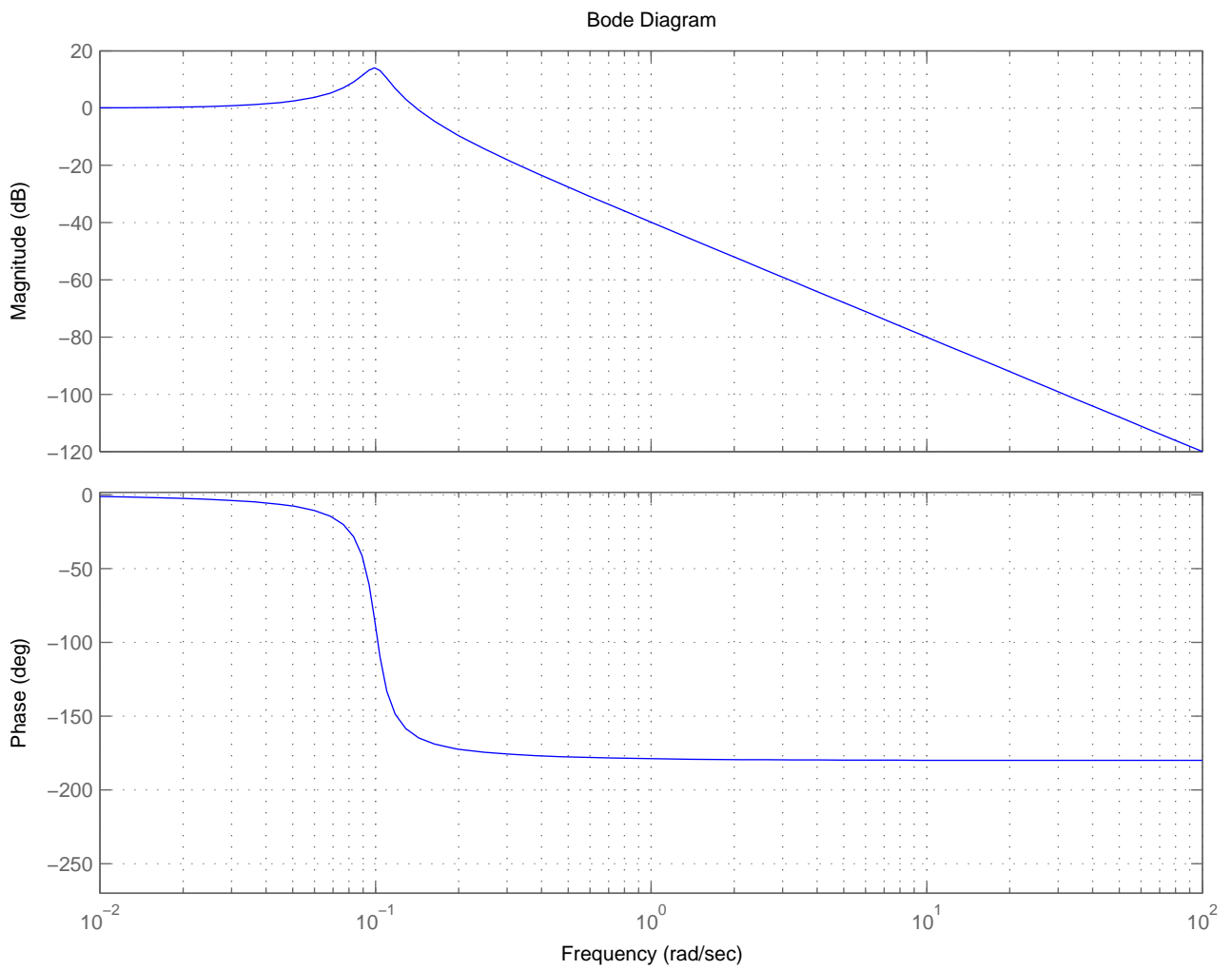


Figure 2: Bode plot for  $G_3$ : draw  $G_1 \cdot G_3$ .

### Bode Diagrams

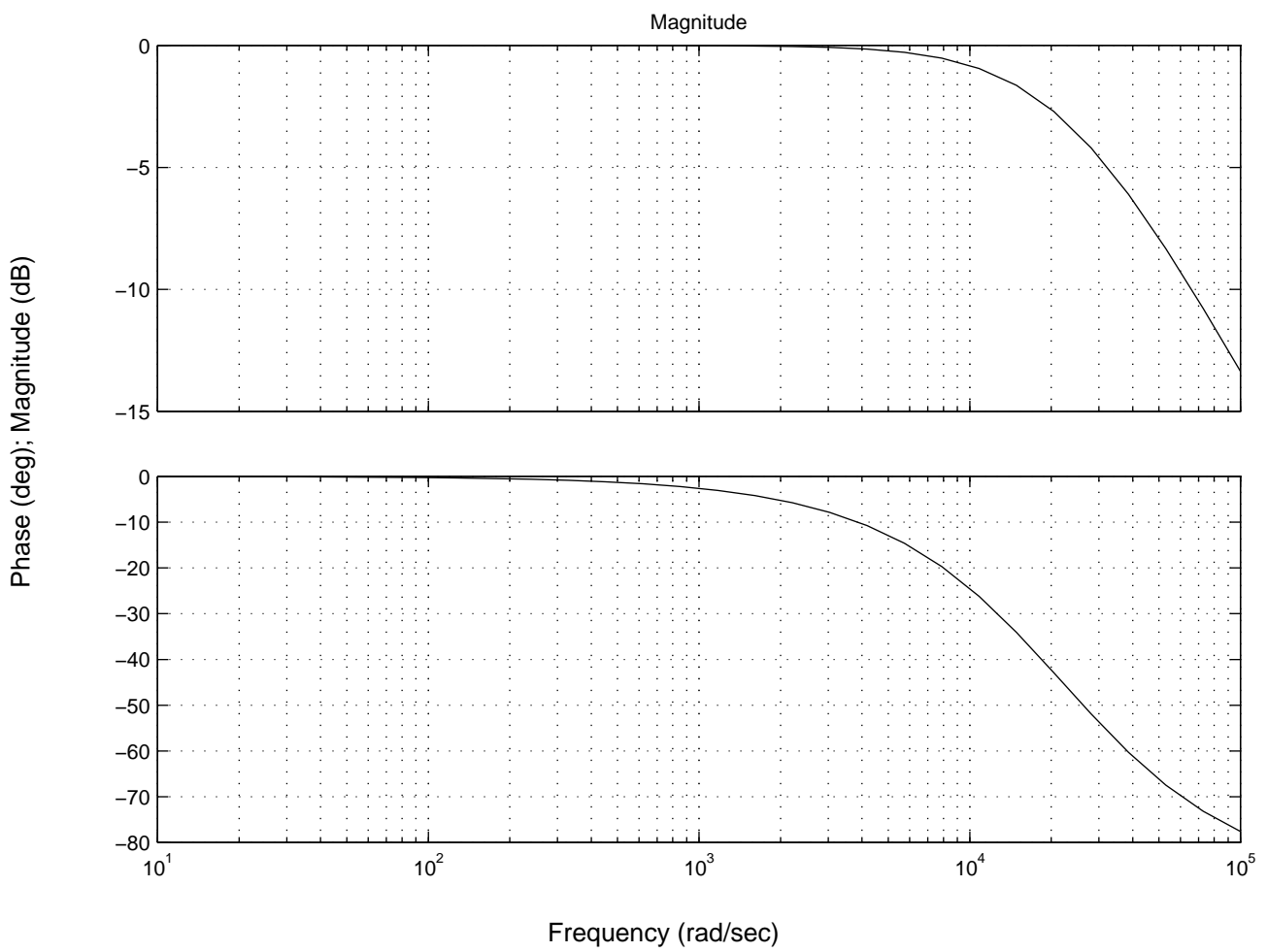


Figure 3: Bode plot for  $H_1(s) = \frac{22000}{22000 + s}$ .

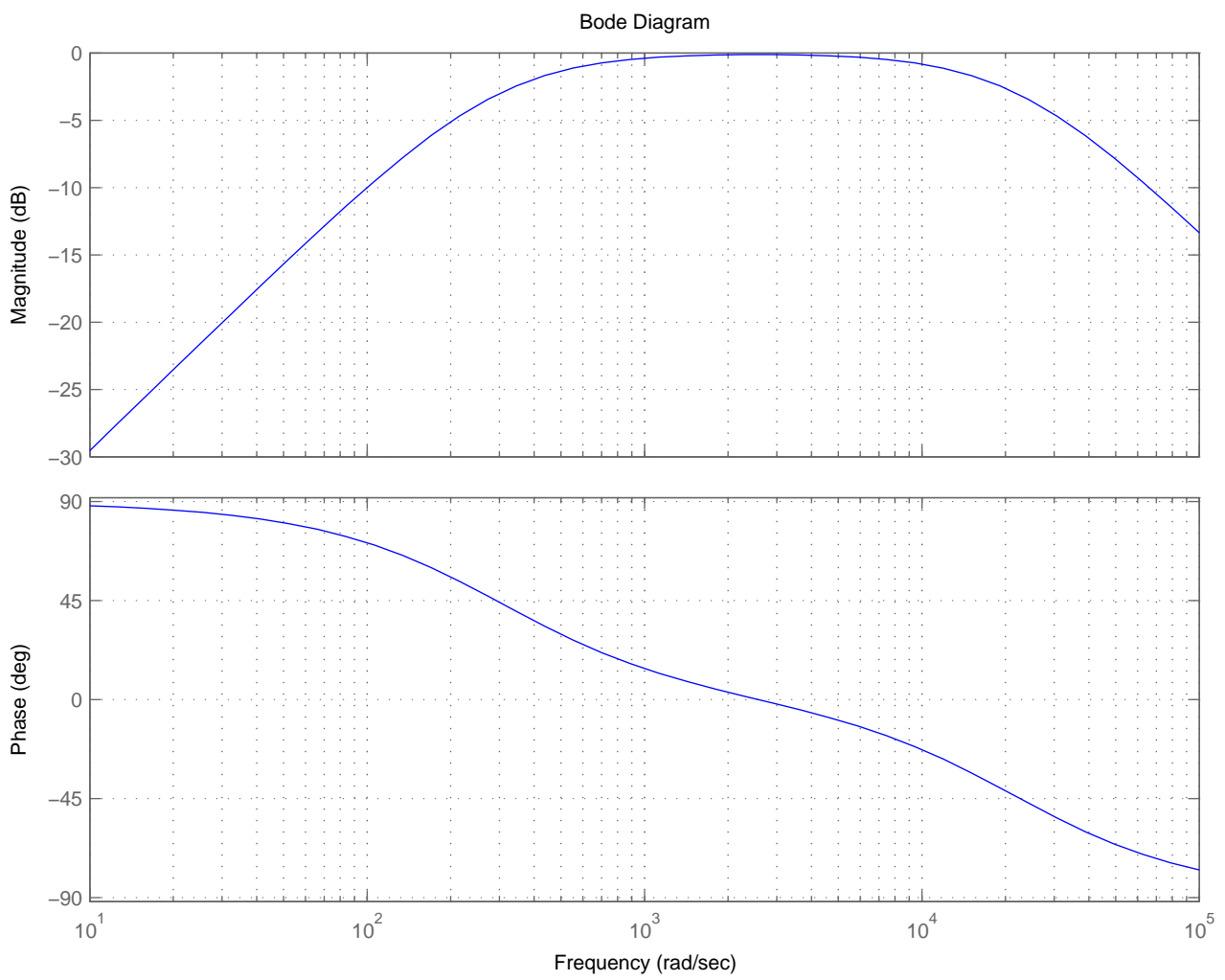


Figure 4: Bode plot for  $H_2(s) = \frac{22000s}{(s+300)(22000+s)}$