Paper 3C1

Examples Sheet 6: Fourier Analysis

Anil Kokaram and Rozenn Dahyot 2003

http://www.mee.tcd.ie/~sigmedia

1. The periodic signal shown in Figure 1 is defined by $x(t) = E\exp(-5t/T)$ in the interval $-0 \leq t \leq T$. Obtain the amplitudes of the d.c. component and the fundamental in this waveform in terms of E. In order to reduce the amplitude of the fundamental the signal is input to the low-pass filter shown. Show that the d.c. component is unaffected by the filter and that the amplitude of the fundamental at the output to the filter is $0.0389E$.

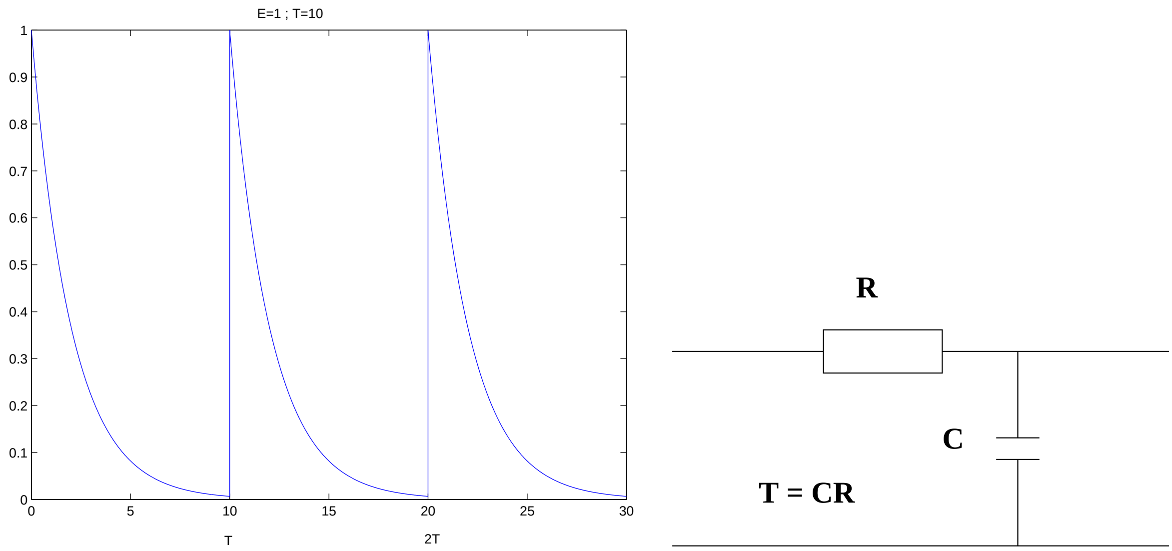

Figure 1: Figures for Question 1.

2. Given that $x(t) \leftrightarrow X(\omega)$ and $y(t) \leftrightarrow Y(\omega)$, prove that $x(t)y(t) \leftrightarrow X(\omega) * Y(\omega)$.

3. Find the Fourier Transform of the following functions given that $f(t) = 0$ for $t < 0$ for the first three functions:

   (a) $f(t) = 10e^{-3t}$

   (b) $f(t) = te^{-t}$

   (c) $f(t) = e^{-2|t|}$

   (d) $f(t) = 2(u(t+1) - u(t-1))$

   (e) $f(t) = 2(u(t+0.5) - u(t-0.5))$

   For the last two functions, sketch $|F(\omega)|$; what would be the effect on the signal spectrum if the width of the pulse were made narrower?

4. Determine the Fourier Transform of the half cosine pulse given by

$$x(t) = \begin{cases} \cos(2\pi t/T) & \text{for } -T/4 \leq t \leq T/4 \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

Using the linearity and shift properties, determine the transforms of the signals shown in Figure 2.

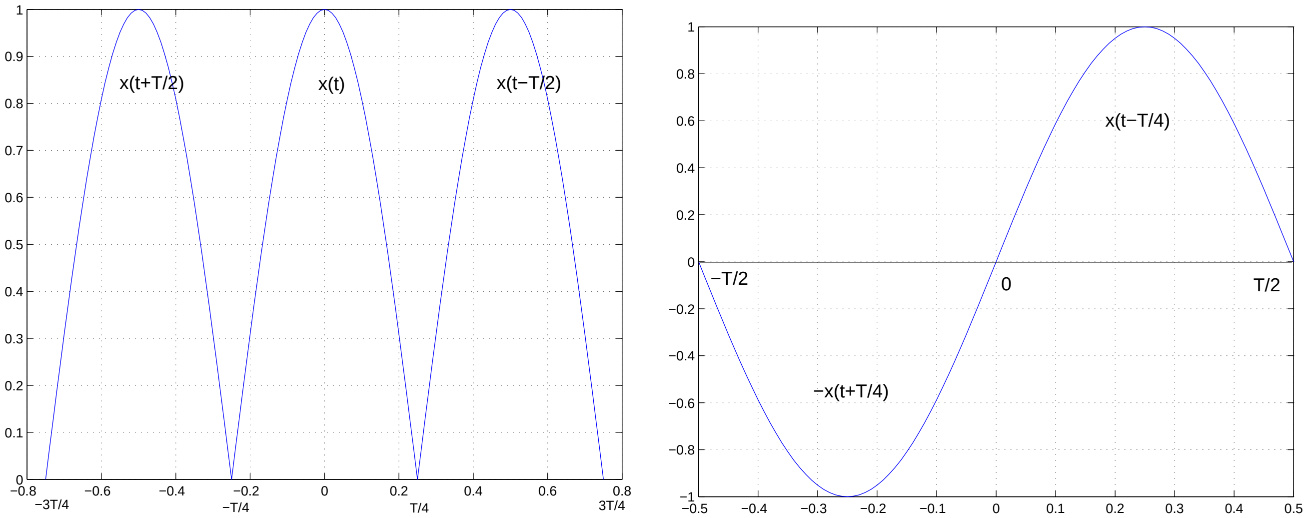

Figure 2: Figures for Question 4.

Answers

1. Amplitude of d.c. and fundamental components are $0.199E$, $0.247E$ respectively.

2. ————

3. (a) $\frac{10}{3+j\omega}$, (b) $\frac{1}{(1+j\omega)^2}$, (c) $\frac{4}{(4+j\omega)^2}$, (d) 4Sinc(ω), (e) 2Sinc($\omega/2$). For (d) and (e), the bandwidth of $f(t)$ increases as pulses become more narrow.

4. (Defining $\omega_0 = 2\pi/T$)

   (a) $\frac{2\omega_0 \cos(\omega T/4)}{\omega_0^2 - \omega^2}$

   (b) $\frac{2\omega_0}{\omega_0^2-\omega^2}(2\cos(\omega T/4) + \cos(3\omega T/4))$

   (c) $\frac{-2j\omega_0}{\omega_0^2-\omega^2}\sin(\omega T/2)$