Paper 3C1

Examples Sheet 7: Digital Systems

Anil Kokaram and Rozenn Dahyot 2003

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Sampling

1. Estimate the minimum sampling frequency that can be used for the following signals if anti-aliasing filters are available with a transition bandwidth that is 20% of their pass bandwidth.
   (a) A strain gauge generating frequencies up to 100 Hz
   (b) A telephone speech signal containing frequencies up to 3400 Hz
   (c) A high quality music signal containing frequencies up to 16 KHz
   (d) A television signal containing frequencies up to 5.5 MHz

2. A signal $x(t)$ has a bandwidth of $\pm B_x$ Hz and a Fourier Transform $X(\omega)$. It is multiplied by a periodic signal $s(t)$ as below
$$s(t) = \sum_{m=-\infty}^{\infty} \delta(t - mT_s) \quad (1)$$
Where T_s is the period of the waveform. The resulting signal, $x_s(t)$, is thus $x_s(t) = s(t)x(t)$.
   (a) Sketch $s(t)$ for $0 \leq t \leq 10T_s$ and find the Fourier Series expansion of $s(t)$
   (b) Derive an expression for $X_s(\omega)$, the Fourier Transform of $x_s(t)$ in terms of $X(\omega)$ and T_s.
   (c) Given that $(1/T_s) = 2.25B_x$, is it possible to recover $x(t)$ from $x_s(t)$? If so, describe how this could be done.

Digital Filters (all examination standard questions)

3. A Digital System has a transfer function given by
$$\mathbf{H}(z) = \frac{0.5(1 + z^{-1})}{1 + 0.2z^{-1} - 0.48z^{-2}}$$
   (a) Draw the system block diagram corresponding to this system.
   (b) Derive an expression for the impulse response of this filter in terms of n, the sample index.
   (c) Assuming zero initial conditions, calculate the first four outputs of the step response of this filter.

4. A Digital System has a transfer function given by
$$\mathbf{H}(z) = 1 + 2z^{-1} + z^{-2}$$
   (a) Write down the difference equation governing this system.
   (b) Given an input sequence $x_n = 0,\ 0,\ 1,\ 1,\ 1,\ 1,\ 1,\ 0, 0, \ldots$, for $n = 0 \ldots 9$, calculate the output sequence when this signal is passed through the digital system above.
   (c) What is the effect of this filter? Is it FIR or IIR? Explain your answer.

Answers

1. 240 Hz, 8160 Hz, 38.4 kHz, 13.2 MHz

2. Bookwork for parts a), b). c) Yes. Use reconstruction filter which is low pass. Need boost at high frequencies if sample/hold operation allowed for. Perfect reconstruction filter is not realisable, but approximation e.g. Butterworth ok.

3. (b) $h_n = \mathcal{Z}^{-1}(\mathbf{H}(z)) = (4/7)(0.6)^n + (-1/14)(-0.8)^n$. (c) $u_0 = 0.5$, $u_1 = 0.9$, $u_2 = 1.06$ and $u_3 = 1.22$.

4. (a) $y_n = x_n + 2x_{n-1} + x_{n-2}$. (b) $y_0 = 0$, $y_1 = 0$, $y_2 = 1$, $y_3 = 3$, $y_4 = 4$, $y_5 =$, $y_6 = 4$, $y_7 = 3$, $y_8 = 1$ and $y_9 = 0$. (c) FIR.