SIGNALS AND SYSTEMS: PAPER 3C1

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- Course is about Signal Analysis and System Analysis. Involves introduction to Digital Signal Processing as well as Control Engineering.
- Both areas use similar techniques but have evolved differently. They are really part of the same study.
- Control engineering is generally the study of techniques for controlling devices/systems/processes. How to keep the temperature of a boiler steady? How to keep the Harrier hovering? How to tune in a radio automatically?
- Signal Processing evolved out of a need to analyse observed phenomenon for instance : to make predictions about the likely state of some system in the future. What will the price of my stock be tomorrow? Can we process this noisy radar echo to work out where the submarine really is? Can I write a computer program that can automatically identify people by pictures of their faces? Can we design a program to remove the 'blotches' from old motion pictures automatically? How can I transmit television to my wireless phone?
- Course begins with Systems Analysis, then an introduction to Digital Signal Processing and finally an introduction to Probability Theory and Random Signals.

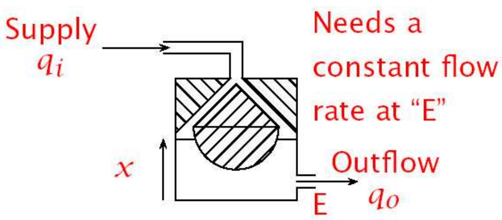
1 Course Outline

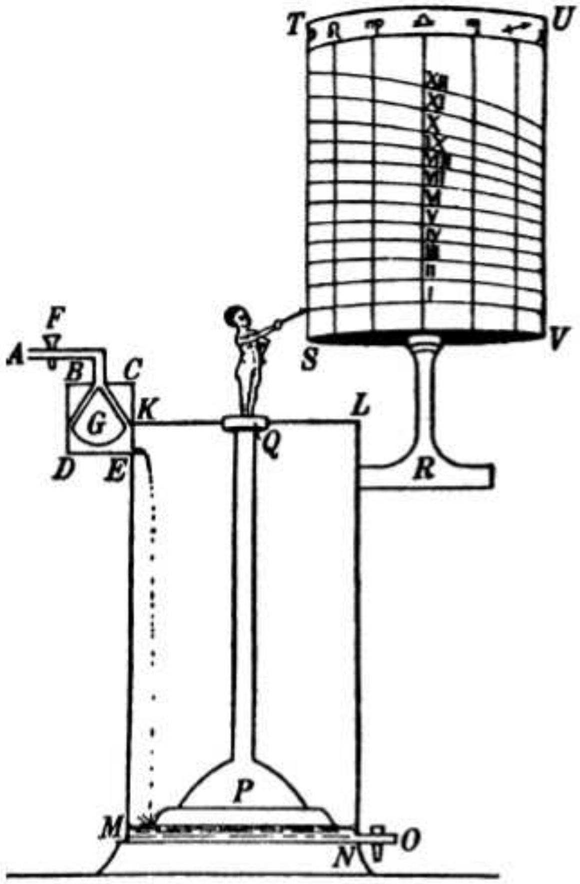
- Two parts. Introduction to Signal and System Analysis 24 L, 9 T; Random Variables 9 L, 3T
- Systems Analysis [Dr. A. Kokaram]
  - Use of block diagrams
  - Differential Equation Models
  - What is a Linear Time invariant System?
  - Impulse response, convolution, step response
  - Laplace Transforms, transfer functions, poles, zeros
  - Stability
  - Fequency response, Steady state response, Low pass and highpass filtering action
- Signal Analysis and Digital Signal Processing [Dr. A. Kokaram]
  - Fourier series, Fourier Transform
  - Sampling theorem, Discrete Signal Processing
  - Low pass filtering, low pass filters
- Introduction to Random Variables [Dr. A Quinn]
  - Probability. What is a random Variable?
  - Introduction to Random Processes. Gaussian, Bernoilli, Poisson distributions
  - Relevance for communications systems
- Recommended Texts
  1. SIGNALS AND SYSTEMS, Oppenheim and Willsky, Prentice Hall 1997 [Avail: Library and Bookshops]
  2. ELECTRONIC SIGNALS AND SYSTEMS, Paul A. Lyn, Macmillan Education (1986?) [Avail: Library]

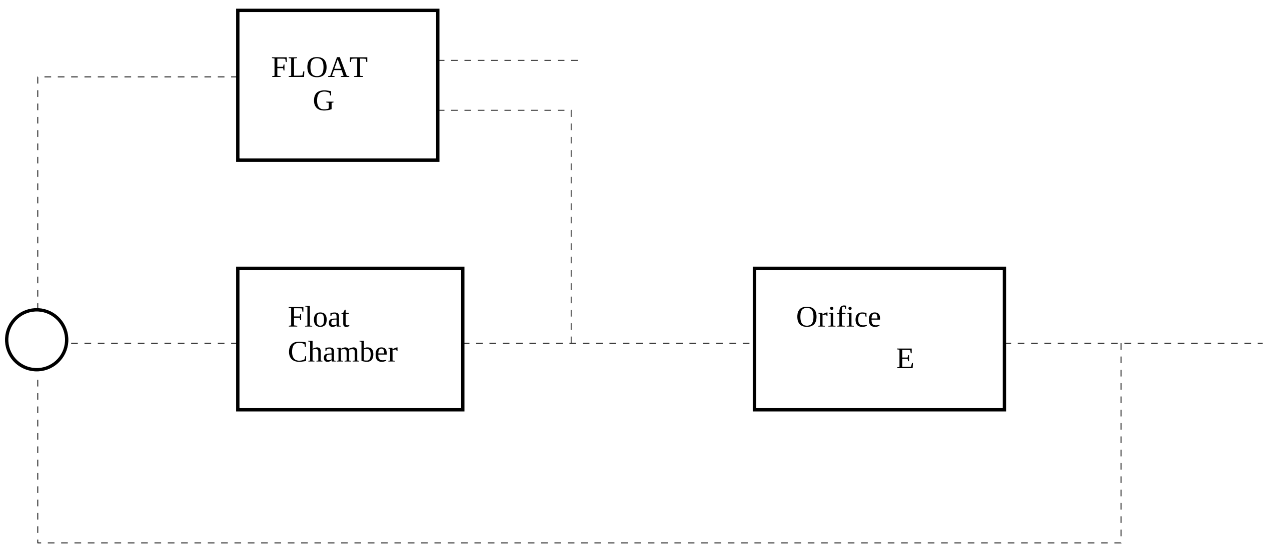
3. MODERN CONTROL SYSTEMS, Dorf and Bishop, Addison Wesley 1998 (8th Ed) [Avail: Library and Bookshops]
4. SYSTEM ANALYSIS AND SIGNAL PROCESSING, Philip Denbigh, Addison-Wesley 1998
5. Note that ordering from `www.amazon.co.uk` is also an alternative to bookshops.

- Web Resources: There are many useful web resources available today that can assist you with this course. We may refer you to some of these.
  - `www.ee.washington.edu/class/235dl` Excellent Java tutorials on convolution, signal manipulation
  - `http://www-dsp.rice.edu/courses/elec301/applets.shtml` Rice University, one of the earliest universities involved with DSP
  - `www.howstuffworks.com` Not a bad place to get a light idea about some systems.
- Need to understand 'big picture' first. Then move on to details.

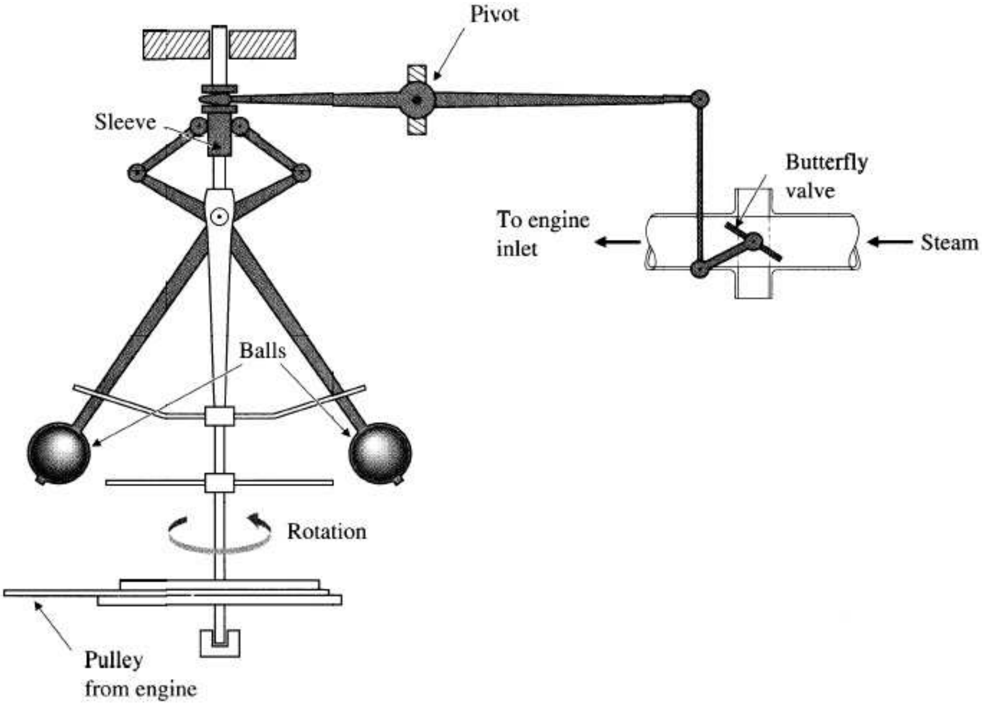
2 A simple (?) system: WATER-CLOCK (Ktesibios, Alexandria 250 BC)

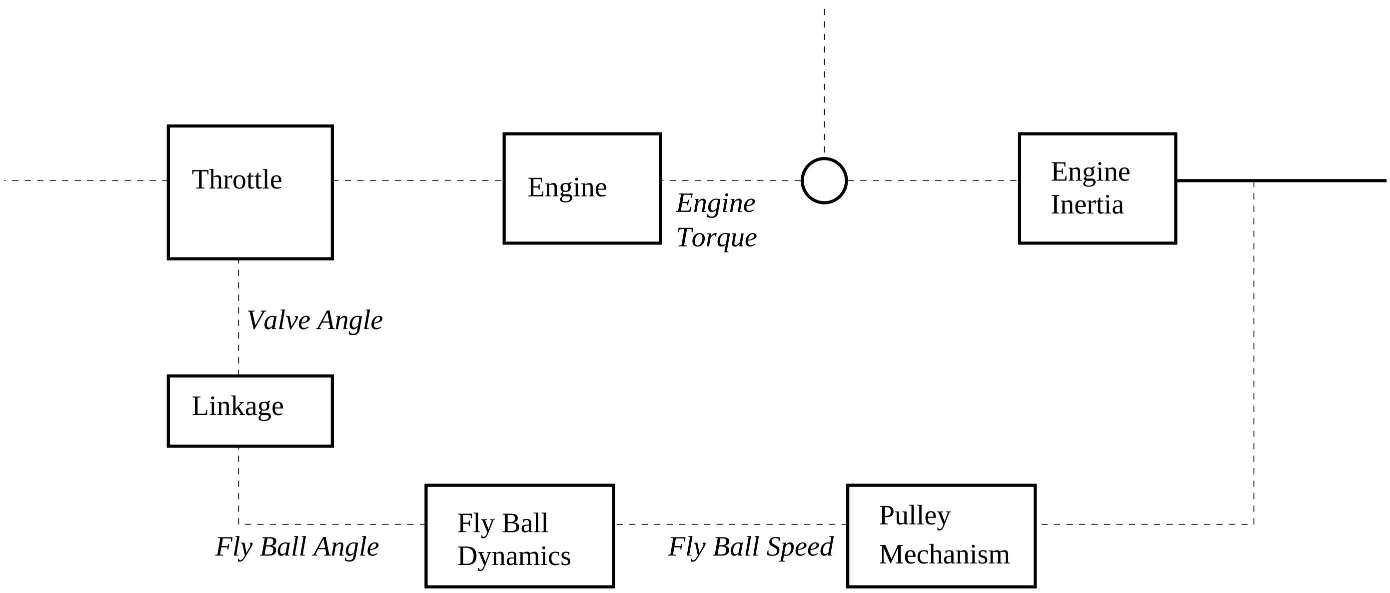




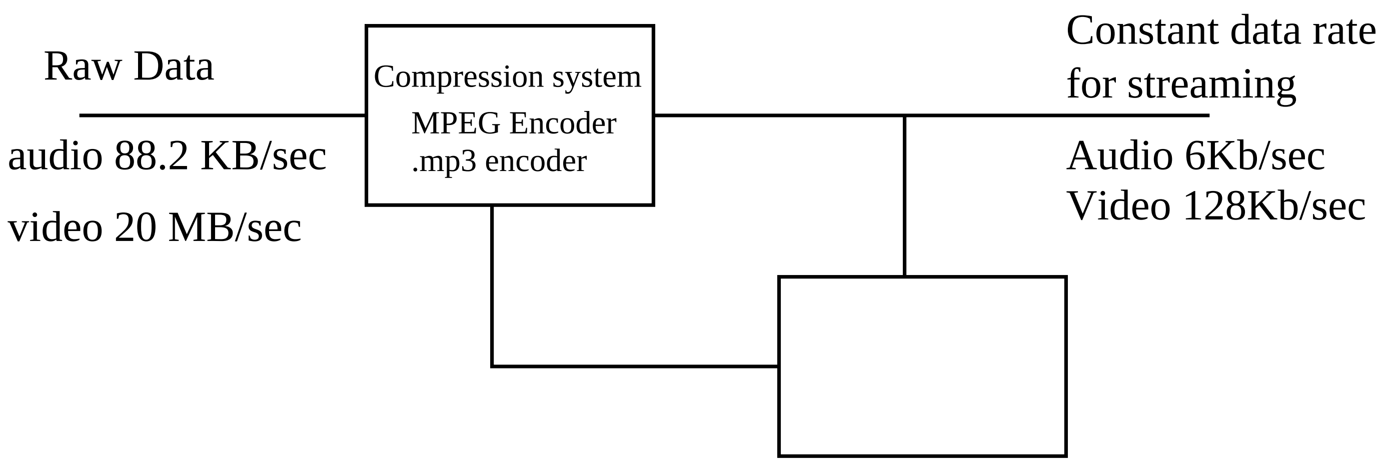


3 A simple (?) system: WATT'S GOVERNOR





4 Rate Control for Streaming Media



5 Signal Analysis

- A signal describes the state of some underlying perhaps 'unobservable' system. It carries information about the system. It may be deterministic or stochastic.

- Deterministic signals are described by 'well behaved' explicit functions. $x(t) = \sin \omega t$. The value of $x(t)$ at any time t is known exactly.

- The state or value of a stochastic signal at any time t can only be determined with some 'probability'. So if $x(t)$ was a binary 'random' variable. One might say that it has value 1 with a probability 0.2 and value 0 with a probability 0.8, say.

- The inflow water rate to the Water clock, for instance, is a stochastic or 'random' signal since the whole point of the control system is to even out the 'uncertainties' in the water flow rate.

- Most real signals are stochastic (random). Speech, images, stock prices. But much harder to analyse. So we'll stick to deterministic ones for the moment.

5.1 Some signals for analysis

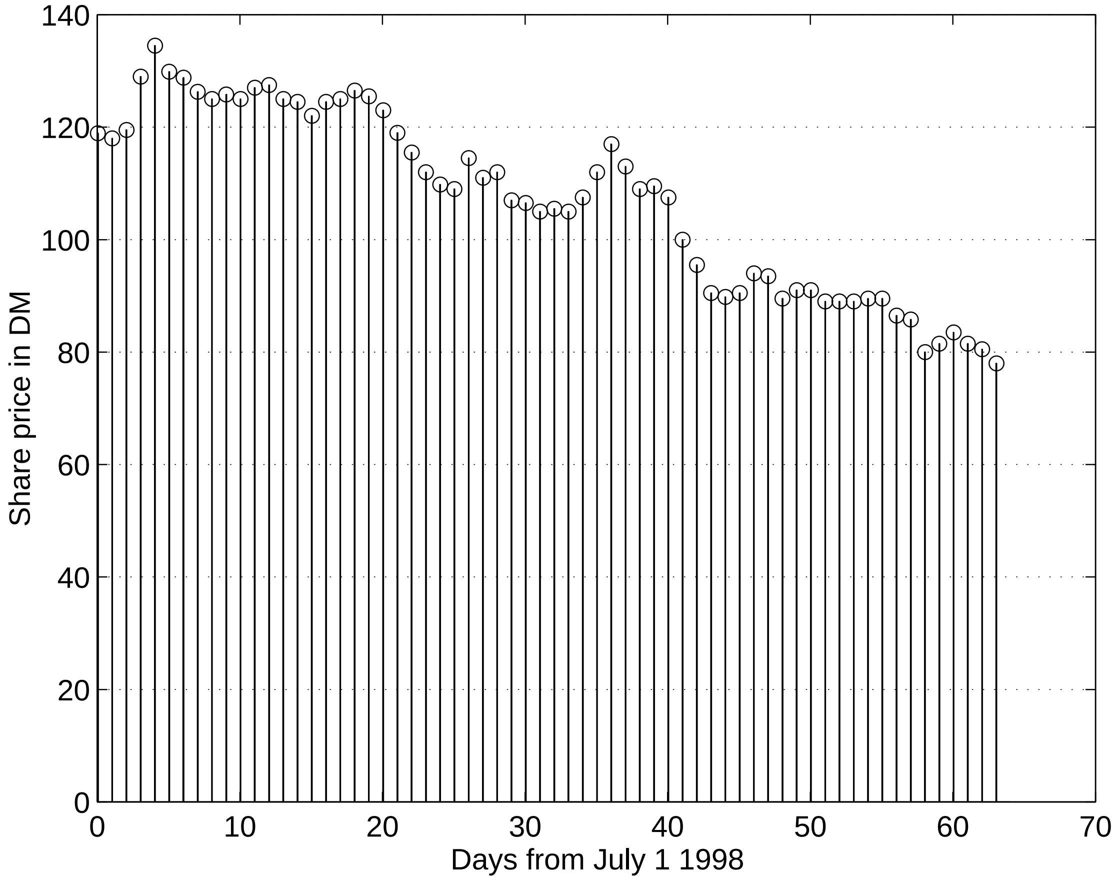
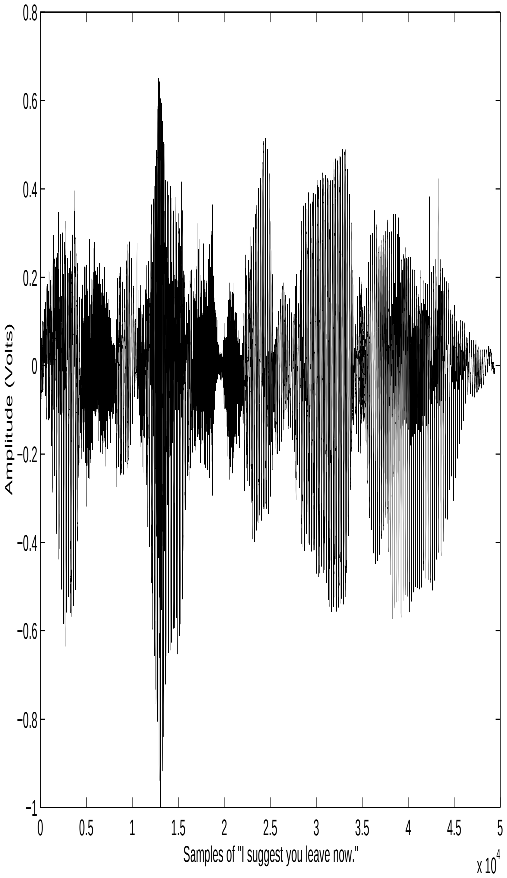
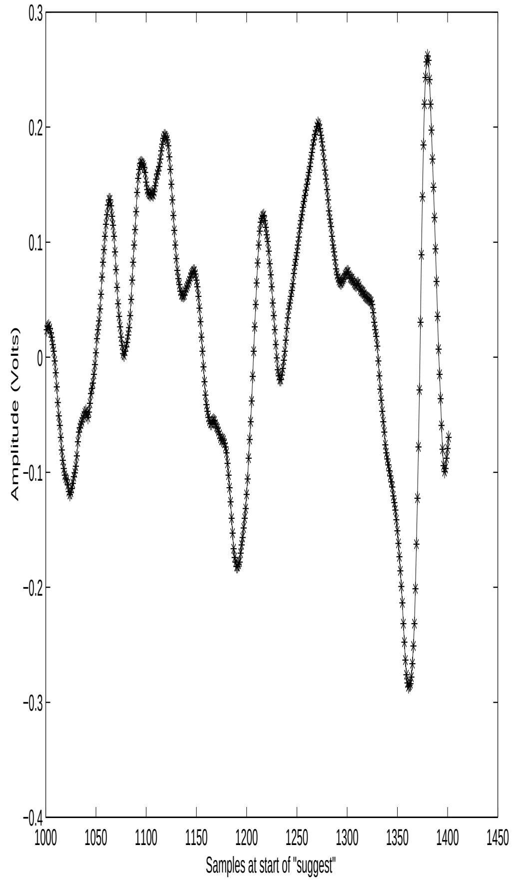


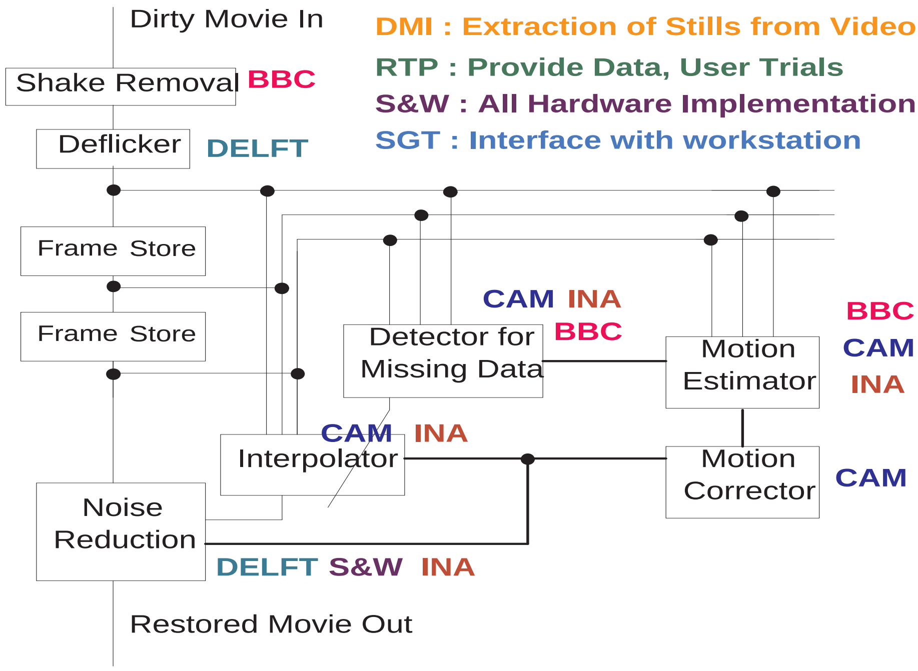
Figure 1: *
Share price at the start of each day since July 1st 1998 for Transtec AG.

Some speech


Amplitude (Volts)
Samples of "I suggest you leave now."
x 10^4


Amplitude (Volts)
Samples at start of "suggest"

A signal processing system: Automatic Motion Picture Restoration



6 SYSTEM ANALYSIS: WHAT HAPPENS IN THE BLOCKS?

Some blocks act like 'amplifiers' or 'attenuators' e.g.

Many are **DYNAMIC** processes described by DIFFERENTIAL EQUATIONS e.g.

O.D.E. Models

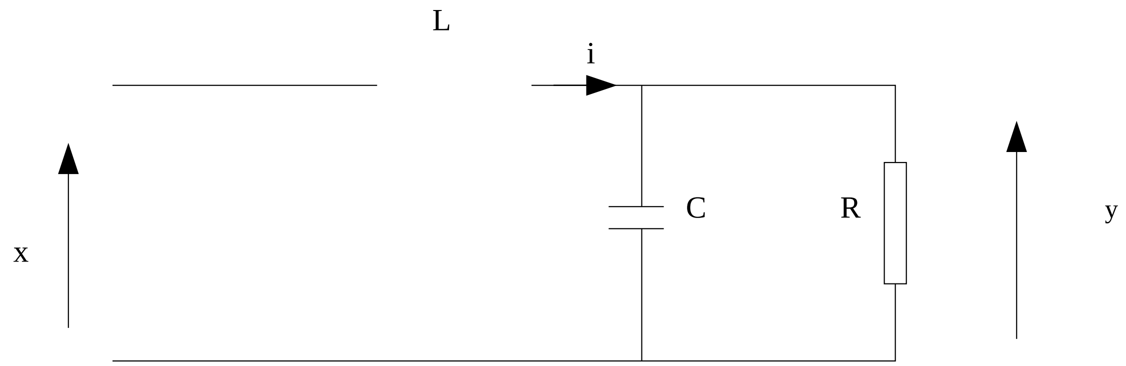


Figure 2: LCR Circuit

Voltage drop around cct.

$$x - L\frac{di}{dt} = y \tag{1}$$

Sum currents at A

$$i = C\frac{dy}{dt} + \frac{y}{R} \tag{2}$$

Differentiate eqn 2 w.r.t. t

$$=$$

Subst into 1 $\Rightarrow$

$$\tag{3}$$

For the control engineer, some blocks are GIVEN e.g

1. Aircraft Dynamics
2. Capacitor behaviour
3. Turbogenerator dynamics

Other blocks have to be designed

1. Geometry of Fly-ball mechanism in Watt Governor
2. Program in the control computer

Note: The sampling rates and ADC/DAC resolutions are often high enough to allow us to pretend that the control computer also bahaves like an ordinary differential equation.

6.1 L.T.I. Systems

This course only deals with LINEAR TIME INVARIANT (L.T.I) SYSTEMS. (Still a large number of systems)

1. A Linear system is one which possesses the important property of *superposition*

   (a) Response to a weighted sum (superposition) of several inputs is a weighted sum of the reponses to each of the inputs.

$$\begin{aligned} \text{IF } x_1(t) &\rightarrow y_1(t) \\ \text{AND } x_2(t) &\rightarrow y_2(t) \\ \text{THEN } x_(t) + x_2(t) &\rightarrow \boxed{\qquad\qquad} \end{aligned}$$

   (b) Response to an input scaled (= multiplied) by any constant is the corresponding output scaled by that same constant.

$$\begin{aligned} \text{IF } x_1(t) &\rightarrow y_1(t) \\ \text{THEN } ax_(t) &\rightarrow \boxed{\qquad} \end{aligned}$$

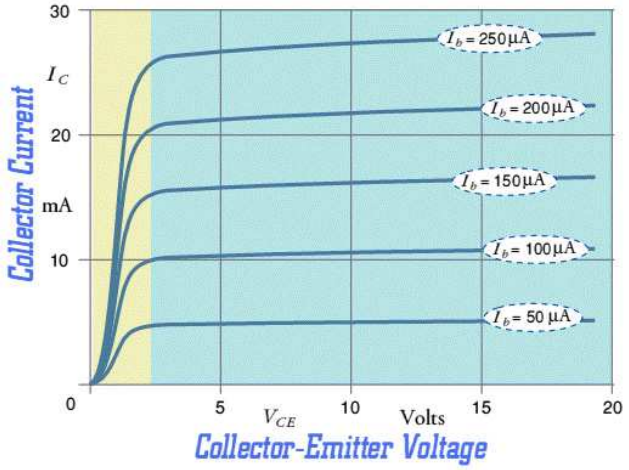
   where a is *any* complex constant. (i.e. a can be either real or complex).

2. Time invariance implies that the behaviour of the system remains the same over time.

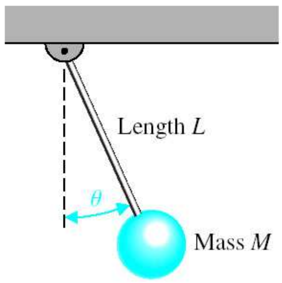
$$\begin{aligned} \text{IF } x_1(t) &\rightarrow y_1(t) \\ \text{THEN } x_(t - \tau) &\rightarrow \boxed{\qquad} \end{aligned}$$

Many systems behave approximately linearly when perturbed slightly

TRANSISTOR



PENDULUM



6.2 Linearisation

O.D.E.'s can usually be 'linearized' around small perturbations about an equilibruim position.

Suppose ODE has the form

$$\dot{y} = f(x, y)$$

and an equilibrium position can be identified as (x_0, y_0), thus at (x_0, y_0) the system is at rest.

$$0 = f(x_0, y_0)$$

Let $x = x_0 + \delta x$ and $\boxed{\qquad\qquad}$, then

$$\dot{y_0} + \dot{\delta y} = f(x_0 + \delta x, y_0 + \delta y)$$

Using Taylor series expansion

$$=$$

Hence, assuming the higher order terms are negligible

$$\dot{\delta y} = \text{CONSTANT} \times \delta x + \text{CONSTANT} \times \delta y$$

which is a Linear ODE!

6.3 Summary

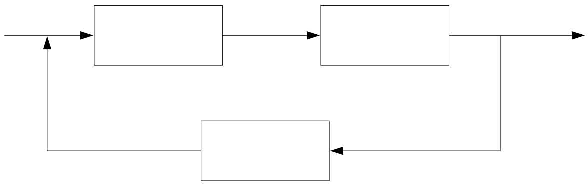


Figure 3: BLOCK DIAGRAMS FOR SYSTEMS

- Each block is a 'SYSTEM'
- We make BIG systems from LITTLE ones
- We concentrate on Causes (inputs) and Effects (outputs)
- We assume systems are dscribed by ODE's
- We assume these ODE's are LINEAR
- We assume all our systems are LINEAR AND TIME INVARIANT

7 Signal definitions and manipulation

A periodic signal with period T secs has a waveform which repeats every T secs.

Aperiodic signals are not periodic.

Can synthesise many useful signals from shifting and summing various rudimentary or 'building block' signals.

These are called the *singularity* functions. They try to quantify discontinuities in signals.

Will show discrete as well as continuous versions of these functions.

7.1 THE STEP FUNCTION

- In continuous time, denoted as $u(t)$. For discrete signals, denoted u_n.

$$u(t) = \begin{cases} 0 & \text{for } t < 0 \\ 1 & \text{for } t \geq 0 \end{cases} \qquad u_n = \begin{cases} 0 & \text{for } n < 0 \\ 1 & \text{for } n \geq 0 \end{cases}$$

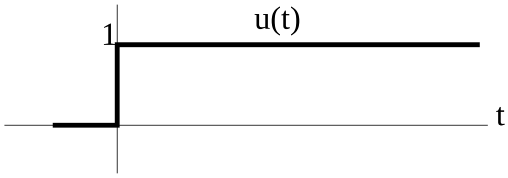


Figure 4: Continuous step

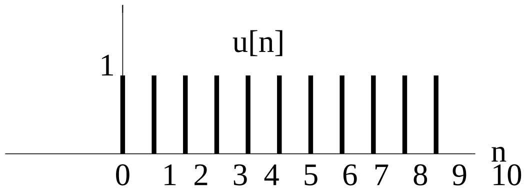


Figure 5: Discrete Step

- $au(t-\alpha)$ is a step of magnitude a occurring at $t = \alpha$. Similarly au_{n-m} is a step of magnitude a occuring at sample number m.

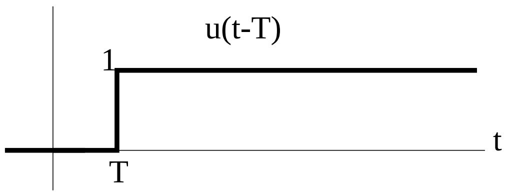


Figure 6: Delayed Continuous step

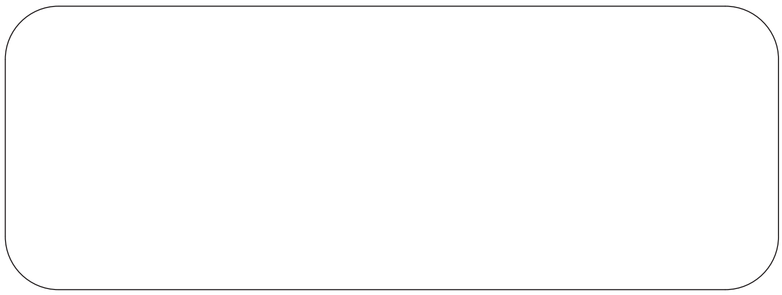


Figure 7: Delayed Discrete Step

- For continuous time, $(t-\alpha)$ **delays** the step by α secs along the time axis.
- For discrete signals, $(n-m)$ **delays** the step by m samples along the time axis.
- Multiplying by a constant (e.g. a above) just scales the step for all time.
- The step function represents one kind of discontinuity that can be found in a signal. The differential of the step function is undefined at the step hence it is 'discontinuous' at the step.

7.2 THE RAMP FUNCTION

- In continuous time, denoted $r(t)$. For discrete signals, denoted r_n.
- It is the integral of the step function.

$$r(t) = \int_\tau u(t)\mathrm{d}\tau \qquad\qquad r_n = \sum_{k=0}^{n} u(n)$$

$$\Rightarrow r(t) = \begin{cases} t & \text{for } t \geq 0 \\ 0 & \text{for } t < 0 \end{cases} \qquad\qquad \Rightarrow r_n = \begin{cases} n & \text{for } n \geq 0 \\ 0 & \text{for } n < 0 \end{cases}$$

- $r(t-\alpha)$ is a ramp occurring at $t = \alpha$. Similarly ar_{n-m} is a ramp occuring at sample number m.

7.3 THE IMPULSE FUNCTION or THE DELTA FUNCTION

- Also called *The Delta function*. Represents a *singularity*.
- The most important primitive function for signals and systems analysis.
- Consider a pulse signal having unit area (can be many shapes). The impulse function is the function which results as the time 'width' of this unit pulse tends to zero.

- The impulse function is denoted $\delta(t)$ for continuous time. It is a purely conceptual device and in fact $\delta(0)$ is undefined.
- $\delta(t-T)$ is a delta or impulse function occuring at $t = T$. The impulse function occurs where its argument is 0.

- It is defined only through its *sifting* property. This means its behaviour is only really defined in context with some signal operation, it does not truly exist as a signal itself.

The *Unit Impulse* is **any** $\delta(t-T)$ which satisfies

$$\int_{-\infty}^{\infty} f(t)\delta(t-T)\mathrm{d}t = f(T)$$

MORE ON THE SIFTING PROPERTY

- Engineers tend to represent the impulse or delta function by an arrow whose height denotes some idea of the 'strength' of the function. This is not a strict mathematical representation, but it allows accurate visualisation of the use of the delta function.

- Therefore, when a signal $f(t)$ is multiplied by an impulse function $\delta(t-T)$ the result is a signal which is zero everywhere except where $t = T$. There its value is the value of the function $f(t)$ at that time $t = T$. This is another delta function with 'strength' $f(T)$. This is another way to think of the sifting property even though it is not strictly correct.

- Some examples.

7.4 THE DISCRETE TIME IMPULSE FUNCTION

- Denoted δ_n
- Same properties as for $\delta(t)$
- Somewhat easier to think about since in discrete time, signals are already broken into 'samples'. Esily visualised as a sample having unit amplitude.
- δ_{n-m} is an impulse at sample $n = m$.
- Obeys *sifting* property.
- $$\boxed{\sum_{n=-\infty}^{\infty} f_n \delta_{n-m} = f_m}$$

8 Revision: COMPLEX NUMBERS

A complex number z can be represented by

$$z = a + \jmath b$$

where
- a is the real part
- b is the imaginary part
- $\jmath$ is the imaginary number $\sqrt{-1}$
- We denote the complex conjugate by z^*

$$z^* = a - \jmath b$$

- The modulus of z is denoted by $|z|$

$$|z| = \sqrt{a^2 + b^2}$$

- The argument of z is given by

$$Arg(z) = tan^{-1}\left(\frac{b}{a}\right)$$

• Viewing the complex number as a vector in the complex plane we see that the modulus is the length of the vector while the argument is the angle it makes with the real axis.

• You should be familiar with the polar form of complex numbers. A complex number z can be written as

$$\begin{aligned} z &= a + \jmath b \\ &= |z|\, e^{\jmath(Arg(z))} \\ &= |z|\, e^{\jmath\theta} \end{aligned}$$

SOME OPERATIONAL LAWS FOR COMPLEX NUMBERS

If

$$z_1 = a + \jmath b$$

and

$$z_2 = c + \jmath d$$

then

$$z_1 + z_2 = (a + c) + \jmath(b + d)$$

$$z_1 \cdot z_2 = (ac - bd) + \jmath\,(ac + bd)$$

Note that

$$|z_1 \cdot z_2| = |z_1|\,|z_2|$$

while

$$\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}$$

Note that

$$Arg\,(z_1 \cdot z_2) = Arg\,(z_1) + Arg\,(z_2)$$

and

$$Arg\left(\frac{z_1}{z_2}\right) = Arg\,(z_1) - Arg\,(z_2)$$

9 Revision: COMPLEX EXPONENTIALS

• These are often used to represent sinusoidal signals. Why ?
• It is easier to manipulate exponential functions than polynomial functions (usually)
• How does this representation work? We use Euler's relation.

$$e^{\jmath\alpha} = \cos\alpha + \jmath\sin\alpha$$

• You can show this equivalence using Taylor series expansions.

Example:

$$\begin{aligned}\cos(\omega t) &= \frac{e^{\jmath\omega t} + e^{-\jmath\omega t}}{2}\\ \sin(\omega t) &= \frac{e^{\jmath\omega t} - e^{-\jmath\omega t}}{2\jmath}\end{aligned}$$

COMPLEX EXPONENTIALS: GRAPHICAL ILLUSTRATION

9.1 Different Sines

DIFFERENT EXPRESSIONS FOR SINE WAVE

So we can express a 50Hz sine wave in several different ways. Let's say that the sine wave has amplitude A volts.

First way: Note that 50Hz means 50 cycles per second. In circular frequency

$$\omega = 2\pi f$$

so

$$\omega = 2\pi 50 = 100\pi$$

Therefore our sine wave is given by

$$x(t) = A\sin(100\pi t)$$

Second way: Or we can say that it is the imaginary part of a complex phasor.

$$\begin{aligned} x(t) &= Im\left[A\cos\theta + \jmath A\sin\theta\right] \\ &= Im\left[Ae^{\jmath\theta}\right] \end{aligned}$$

where $\theta = \omega t = 100\pi t$

Third way: Or we can express it as the sum of two complex phasors.

$$x(t) = \frac{Ae^{\jmath\omega t} + Ae^{-\jmath\omega t}}{2\jmath}$$

10 Basic signal operations

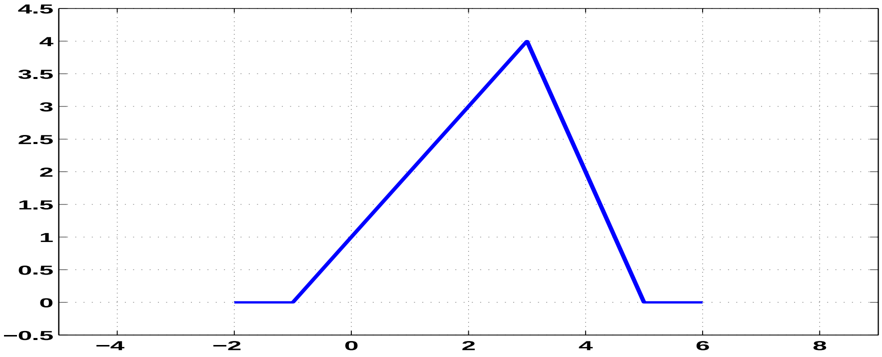
1. Amplitude scaling
2. Time shift.
3. Combination of scaling and shifting
4. Synthesising pulse signals

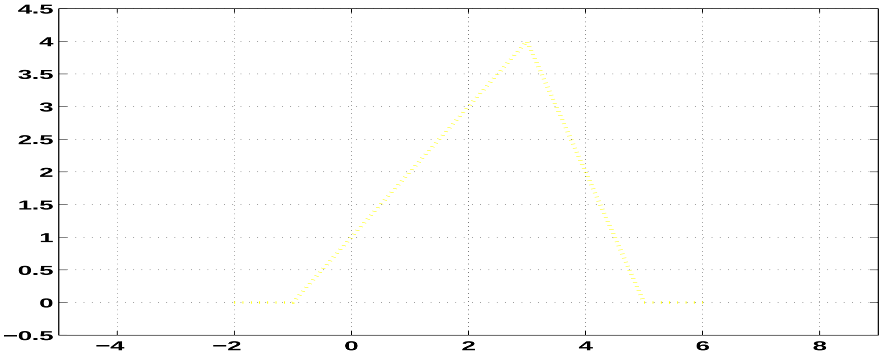
10.1 Amplitude Scaling and Addition

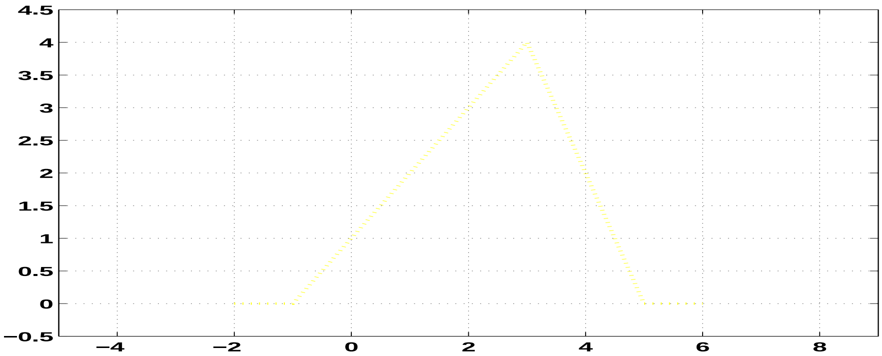
Given $x(t)$ as below, sketch $3x(t)$ and $3x(t) - 1$.

10.2 Time shift

Given $x(t)$ as below, sketch $x(t-3)$ and $x(3-t)$

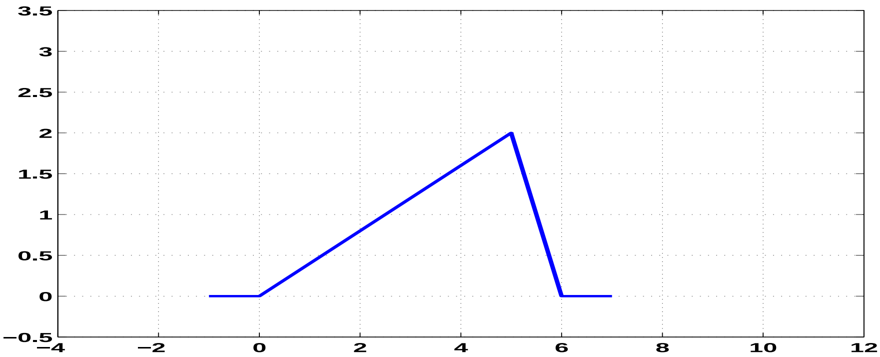


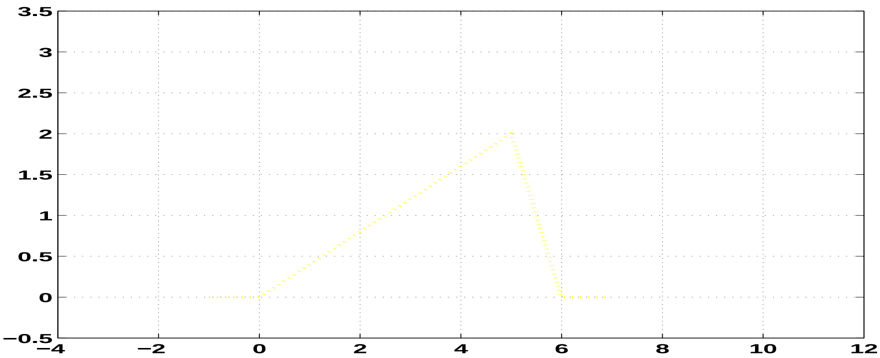


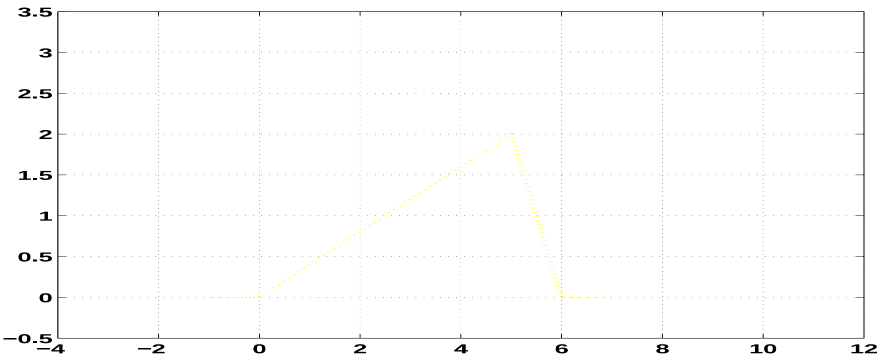


10.3 Combination of time scaling and shifting

Given $x(t)$ as below, sketch $x(2t)$, $x(t/2)$, $x(2t-3)$







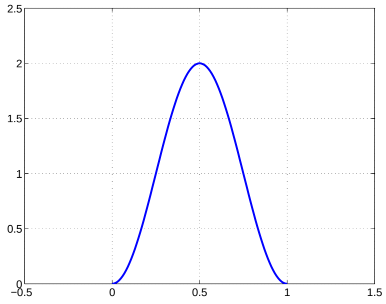
10.4 Synthesising signals

1. Express $p(t)$ (given below) in terms of $u(t)$, the step function.

$$p(t) = \begin{cases} 0 & t < 0 \\ 2 & 0 \leq t \leq 3 \\ 0 & t > 3 \end{cases}$$

2. Construct $p(t)$ (given below) using cosines and the step function.

$$p(t) = \begin{cases} 0 & t < 0 \\ 1 + \cos(2\pi(t - \frac{1}{2})) & 0 \leq t \leq 1 \\ 0 & t > 1 \end{cases}$$



SUMMARY

- Can synthesise many signals using combinations of primitive signal forms.
- Step function at $t = T$ is $u(t - T)$.
- Ramp function is integral of step function.
- Impulse function is simplest primitive. Has a 'sifting' property.
- When impulse function is multiplied by a signal result is the value of the signal at the location of the impulse. (Remember this is not strictly true since integration is the only operation that $\delta(t)$ is defined with. But this 'truth' is good enough for us.)
- *Delay* means shifting signal to 'right'. Advance means shift signal to 'left'.
- 'Time reversal' operation causes reversal in signal direction. A 'mirror' image of signal results.

YOU SHOULD NOW BE ABLE TO DO ALL THE QUESTIONS ON EXAMPLE SHEET 1 AND 2