UNIVERSITY OF DUBLIN

TRINITY COLLEGE

FACULTY OF ENGINEERING & SYSTEM SCIENCES

Department of Electronic and Electrical Engineering

Junior Sophister **Trinity Term, 2010**

SAMPLE Engineering Examination

Part I

Signals and Systems (3C1) SAMPLE PAPER

Friday XX XX^{th} **SAMPLE PAPER** **14:00-16:00**

Prof. W. Coffey and Prof. A. C. Kokaram

Instructions to candidates:

Answer FOUR (4) questions. You must answer at least one question from each section.

Start your answer to each question on a new page.

Materials permitted for this examination:

- **Calculator**
- **Log Tables**
- **Standard Trinity College Book of Equations/Tables**
- **Tables for this course as supplied with this question sheet**
- **Ruler**
- **Manuscript Paper**

Section A

1. A system, G_1 has transfer function as follows.

$$\mathbf{G}_1(s) = \frac{1}{s^2 - 2s\sqrt(2) + 4}$$

(a) The system is controlled by a negative feedback process as shown in figure 1. K is a constant used to control the overall system, and $G_2(s) = (s+2)$. Write down the expression for the new overall system transfer function $\frac{\mathbf{Y}(s)}{\mathbf{X}(s)}$.

(b) For what range of values for K is the overall feedback system stable?

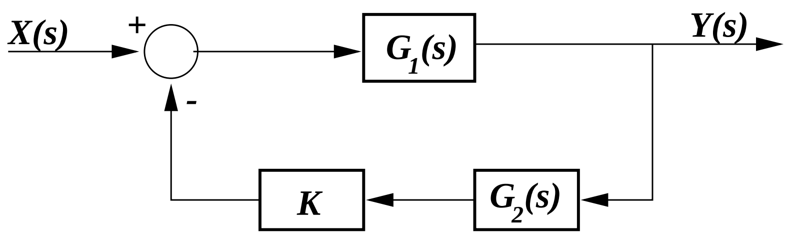

Figure 1: For question 1.

2. Figure 2 shows the magnitude response for two systems G_1 and G_2. The system transfer functions are as follows.

$$G_1(s) = \frac{(3000\pi)}{(3000\pi) + s}; \quad G_2(s) = \frac{1}{1 + \frac{2\times.561}{6000\pi}s + \frac{1}{(6000\pi)^2}s^2}$$

   (a) Match the systems with the magnitude responses shown in the figures and explain your answer.

   (b) Given the periodic signal $x(t)$ as below in figure 2 (with $T = 1/500$ secs), calculate the magnitude of the spectral components of the signal $x(t)$ at frequencies $f = 500,\ 10^3,\ 10^4$ Hz.

   (c) $x(t)$ is presented to the input of the system $G_1(s)$. The output signal is $y(t)$. Using the magnitude response curves in figure 2, calculate the magnitude of the spectral components of the signal $y(t)$ at frequencies $f = 500,\ 10^3,\ 10^4$ Hz.

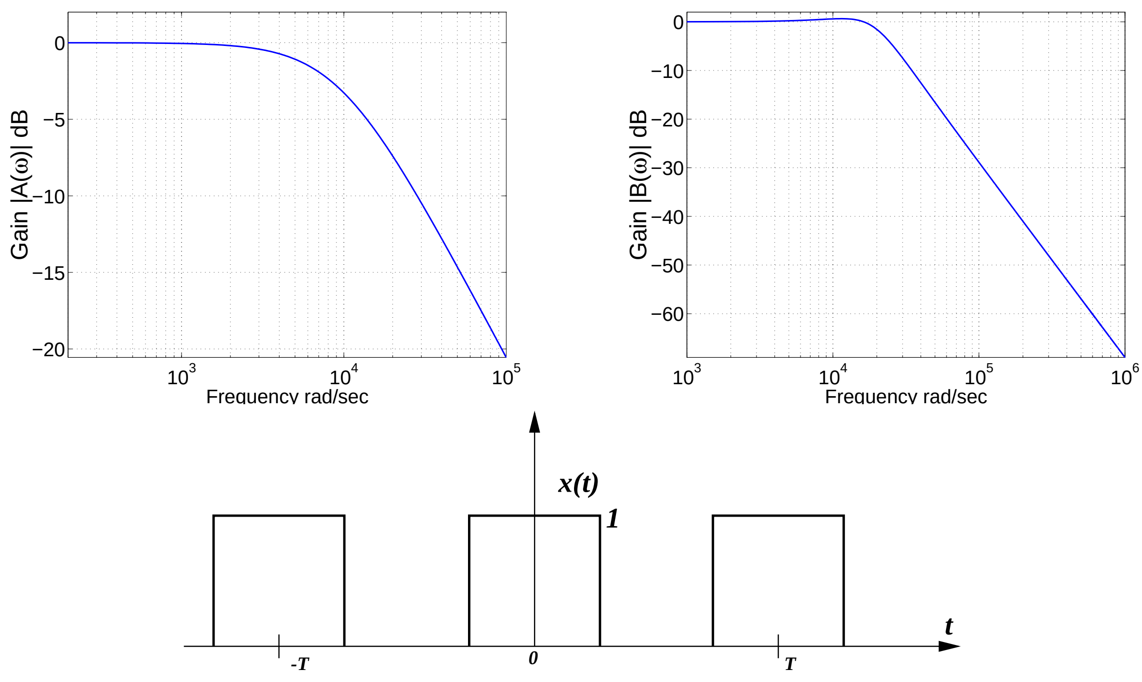

Figure 2: For question 2. Top Left: Gain $|A(\omega)|$, Top Right: $|B(\omega)|$. Bottom: Signal $x(t)$

3. (a) A periodic signal $s(t)$ is defined as follows

$$s(t) = T \sum_{k=-\infty}^{\infty} \delta(t - kT)$$

where T is the period of the signal. Sketch the signal $s(t)$ and derive an expression for its Fourier series expansion.

(b) A signal $x_s(t)$ is created from an audio signal $x(t)$ by $x_s(t) = s(t)x(t)$. Show that the spectrum of $x_s(t)$, $X_s(\omega)$ is given by the expression below.

$$X_s(\omega) = \sum_{n=-\infty}^{\infty} X(\omega - n\omega_s)$$

where $\omega_s = 2\pi/T$, and $X(\omega)$ is the Fourier transform of $x(t)$.

(c) Given $x(t) = \sin(\omega t)$ where $\omega = 2\pi f$ with $f = 1000Hz$, sketch $|X_s(\omega)|$ for the interval $-2\pi(2/T) < \omega < 2\pi(2/T)$ when $T = (1/1000),\ (1/3000)$ secs.

4. Given a digital system governed by a difference equation as follows

$$y_n - 0.94y_{n-1} + 0.33y_{n-2} = 0.1x_n + .2x_{n-1}$$

(a) Write down an expression for the system transfer function $H(z) = Y(z)/X(z)$ of the digital system.

(b) Draw the block diagram representing this system.

(c) Given the signal $x_n = 1,\ 2,\ 1,\ 0$ for $n = 0, 1, 2, 3$, and given x_n is zero otherwise, calculate the first three outputs from this digital system when it is used to filter the signal x_n.

5. Two digital systems $H_1(z)$ and $H_2(z)$ have system transfer functions as follows

$$H_1(z) = [1 - 1.8\cos(\pi/6)z^{-1} + 0.81z^{-2}]$$
$$H_2(z) = \frac{1}{\left(1 + z^{-1}0.99e^{\left(-j\frac{\pi}{6}\right)}\right)\left(1 + z^{-1}0.99e^{\left(j\frac{\pi}{6}\right)}\right)}$$

(a) What type of filters are H_1 and H_2?

(b) Derive expressions for the impulse response for both systems.

(c) A signal x_n shown in figure 3 is processed separately with both systems, yielding outputs a_n and b_n shown in the bottom part of figure 3. Match the systems to the outputs giving your reasons.

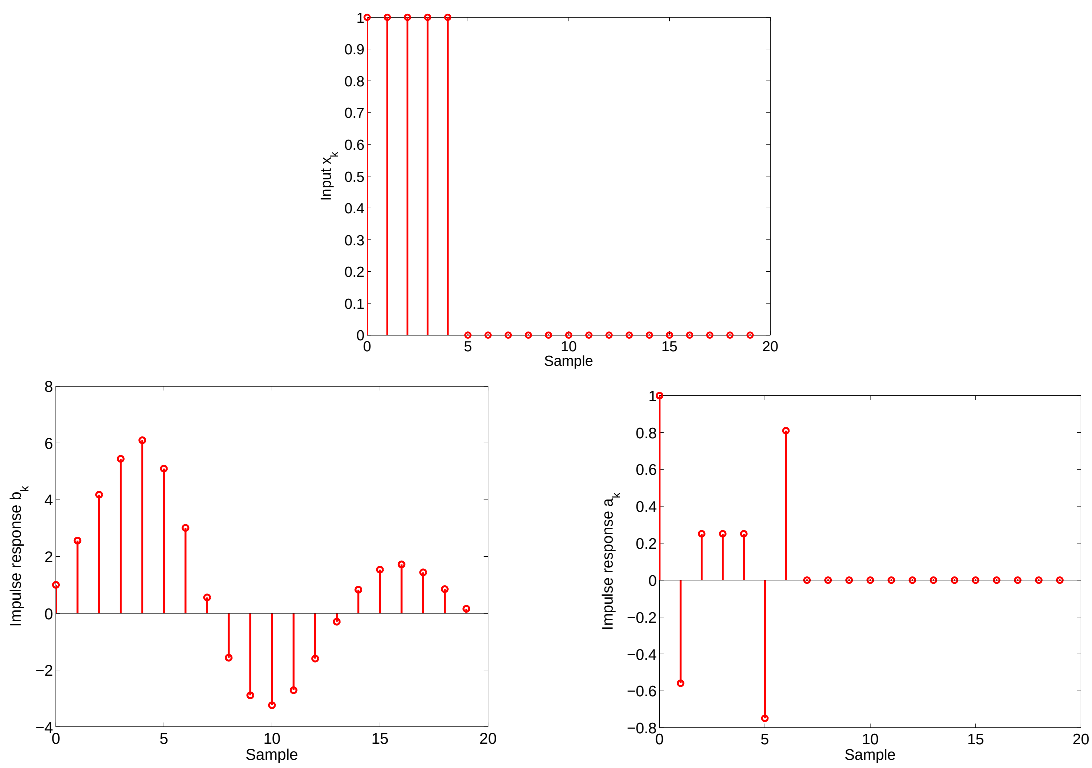

Figure 3: For question 6. Top: Signal x_n, Bottom: Output when x_n is processed by the systems, left a_n, right b_n. The horizontal axis is the sample index n.

Section B

6. (a) Show from first principles that the autocorrelation function of a stationary stochastic process is the Fourier transform of the spectral density.

   (b) A particular oscillating system is governed by the second order, linear differential equation

$$\frac{d^2}{dt^2}y(t) + \beta\frac{d}{dt}y(t) + \omega_0^2 y(t) = f(t)$$

   Find the transfer function $\chi(\omega) = Y(\omega)/F(\omega)$ where the capitals denote Fourier transforms. Hence write down in integral form the output spectral density if $f(t)$ is white noise.

7. (a)) What is meant by the terms autocorrelation function, time average, ensemble average, spectral density, variance and covariance as applied to a stationary stochastic process?

   (b) Write down the autocorrelation function (acf) for white noise. Hence obtain the spectral density of (a) white noise and (b) white noise passed through a filter with transfer function $\chi(\omega)$ (below) and determine the acf of the filtered noise.

$$\chi(\omega) = \frac{1}{1 + i\omega\tau_0}$$