Paper 3C1

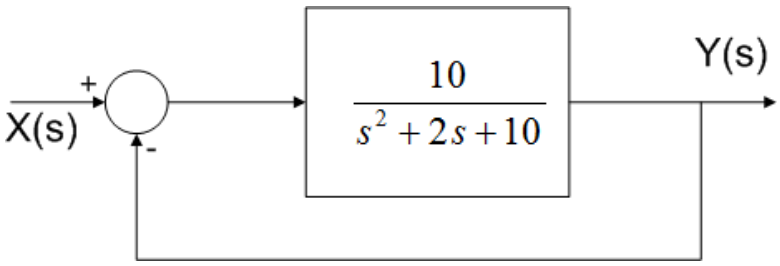
Examples Sheet 8: Control Systems

David Corrigan 2011

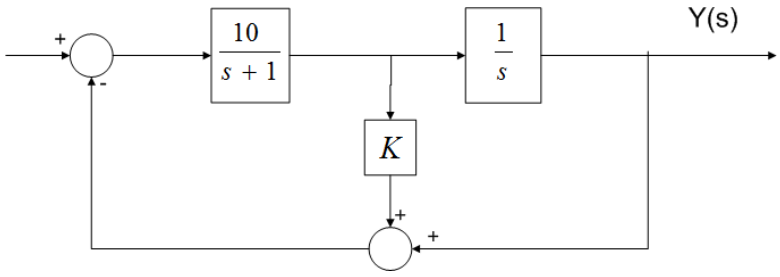
http://www.mee.tcd.ie/~sigmedia

1. Estimate the transfer function $H(s) = Y(s)/X(s)$ for each of the following control systems.

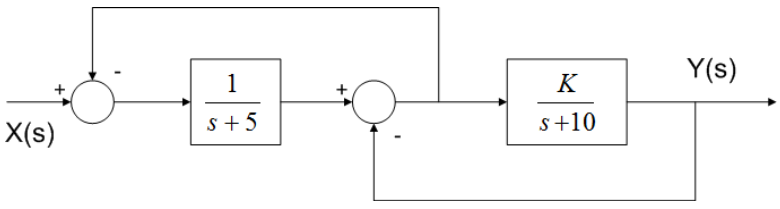
(a)



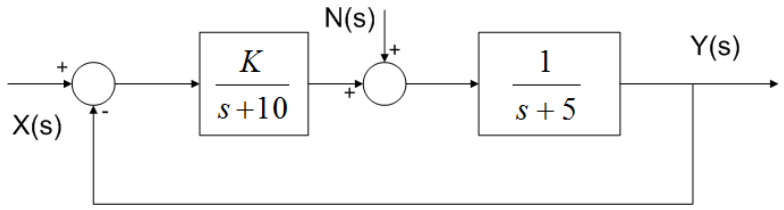
(b)



(c)



2. Consider the following closed-loop system



   (a) Find the system transfer function $H(s) = Y(s)/X(s)$.

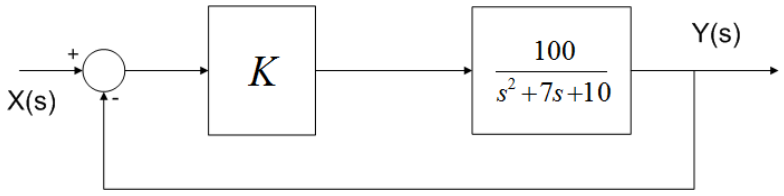
   (b) Calculate the steady-state error due to a unit step input.

   (c) Calculate the steady-state error due to a unit ramp input.

   (d) Find the disturbance transfer function $F(s) = Y(s)/N(s)$.

   (e) Calculate the steady-state error due to a unit step disturbance.

3. Consider the following control system



   For a unit step input find the gain that gives a steady state error of 1%.

   For that gain find

   (a) the settling time to a 2% tolerance.

   (b) the percentage overshoot.

4. A closed-loop system has two dominant complex conjugate poles and no finite zeros. Sketch the region of the s-plane where the poles should be located so that the system will be stable and the following specifications are met.

   (a) $0.5 < \zeta < 1/\sqrt{2}$ and $\omega_n > 1$.

   (b) $\zeta < \sqrt{3}/2$ and $\omega_n\sqrt{1-\zeta^2} < 1$

   (c) A 2% settling time < 2 and a percentage overshoot of $< 50\%$.

   (d) An overshoot of less than 50% and a peak time of less than 2 seconds.

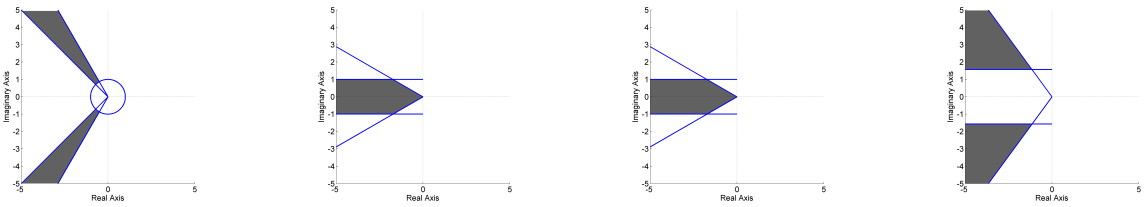
5. For a closed-loop system shown below plot the root locus for $0 \leq K < \infty$

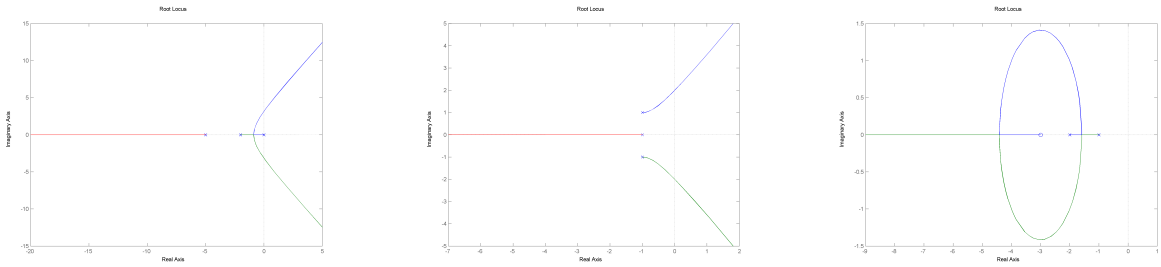
   (a) $C(s)G(s) = \frac{K}{s(s+5)(s+2)}$

   (b) $C(s)G(s) = \frac{K}{(s^2+2s+2)(s+1)}$

   (c) $C(s)G(s) = \frac{3(s+K)}{s^2+Ks+2}$

Answers

1. (a) $\frac{10}{s^2+2s+20}$, (b) $\frac{10}{s^2+(10K+1)s+10}$, (c) $\frac{K}{s^2+(K+16)s+60+5K}$.
2. (a) $\frac{K}{s^2+15s+50+K}$, (b) $\frac{50}{50+K}$, (c) ∞, (d) $\frac{s+10}{s^2+15s+50+K}$, (e) $\frac{10}{50+K}$.
3. $K = 9.9$ (a) 1.143, (b) 70.5%.
4. 

(a) (b) (c) (d)
5. 

(a) (b) (c)